AL-ANBAR University
College of Engineering
Dams and Water
Resources Eng. Department
4th Class

DESIGN OF RENFORCMENT & HYDRULIC CONCRETE STRUCTURE

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Design of Reinforced Concrete Structures
تصاميم المنشآت الخرسانية المسلحة

المصادر :-

- أساسيات الخرسانة المسلحة :- د. سعد علي الطعان - جامعة الموصل
- تصاميم الخرسانة المسلحة بموجب الكود الأمريكي ACI 318M-95

- Design of Reinforced Concrete Structures by Winter
- تصاميم المنشآت الخرسانية المسلحة وفقاً لمتطلبات الكود (ACI 318-02) لمؤلفه الدكتور جمال عبدالواحد فرحان الظاهر - قسم الهندسة المدنية - كلية الهندسة - جامعة الأنبار

* مكونات الخرسانة

Cement + Aggregate + Water + [Some times Admixtures]

مقاومة الخرسانة

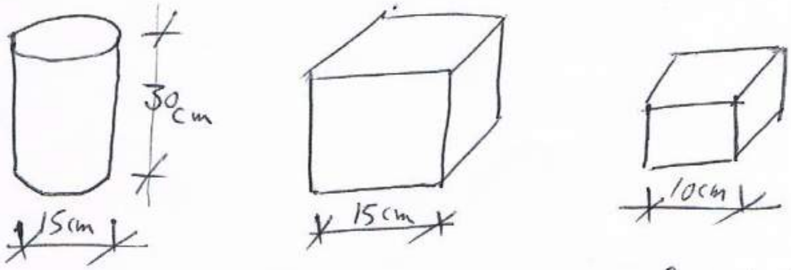
* مقاومة الانضغاط Compressive Strength

* Compressive Strength is indirect indicator for other properties

* Good Comp. St. ⟹ Good Concrete

* 28-day is the age which is used to test concret samples for finding design compressive strength.

* Specimens which are used to find Comp. St. :-



American Standard   B.S. Stand.   for lab. Uses

Tensile Strength مقاومة الشد

- Tensile Strength is so small when compare with Comp. St. & it is about (10-15)% of Comp. St.

Shear Strength مقاومة القص

- Varied between (35-80)% of Comp. St.

Types of Concrete أنواع الخرسانة

- Calssification of concrete according to unit weight.

- Normal weight Conc. : 2300 kg/m^3 to 2400 kg/m^3

  (2300 kg/m^3 ← without Steel Reinf.; 2400 kg/m^3 ← with Steel Reinf.)

- Light weight Conc. : It's density less than 1800 kg/m^3

* Made from natural lightweight aggregate or industrial lightweight aggregate.

* No-fines Concrete.

* Foamed Concrete & Cellural concrete.

- Heavy weight concrete: Made from natural Iron Ore, which are crushed to small sizes, for use as aggregate. $\gamma = (3200 - 4000)\ kg/m^3$

• Classification of concrete according to Comp. St.

* Low St. Conc. / Comp. St. < 20 MPa
* Medium St. Conc. / 20 MPa < Comp. St. < 60 MPa
* High St. Conc. / Comp. St. > 60 MPa

• There are different other types of Concrete like :- Polymer Concrete, Fiber Reinforced Conc., Expansive-Cement Conc., Self Compacted Conc., Roller compacted Conc., ... Etc.

Type of Reinforcement : Steel Reinforcement حديد التسليح *

- Reinforcing Bars
- Welded Wire Fabric
- Prestressing Steel

* $\frac{1}{100}$ of Conc. Volume
* High in cost

Flexural Analysis of Beams by Working Stress Method

Loads :- Wind Loads , Dead Loads, Live Loads

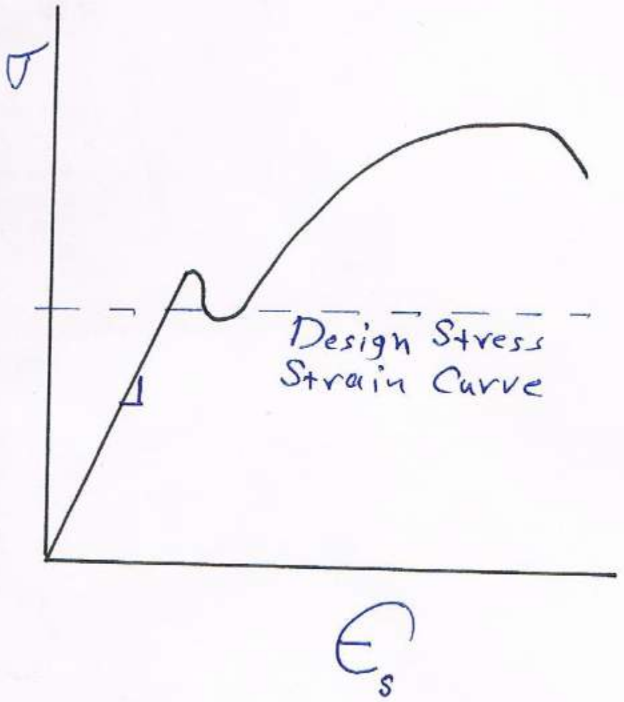
Dead Loads: (kN/m^2); Live Loads: (kN/m^2)

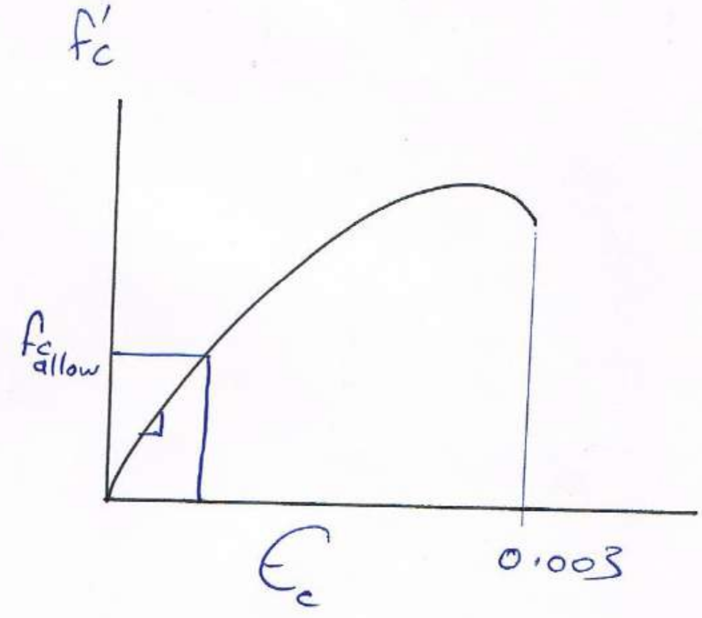
Wind Loads ↙

$$q = kV^2$$

(V = سرعة الريح (km/hr) / k = ثابت)

- Using of real loads $W = L + D$
- Using parts of Materals Strengths





$$E_c = 4700\sqrt{f'_c}$$

أعلى اجهاد شد تتحمله الخرسانة f_r

$$f_r = 0.7\sqrt{f'_c}$$

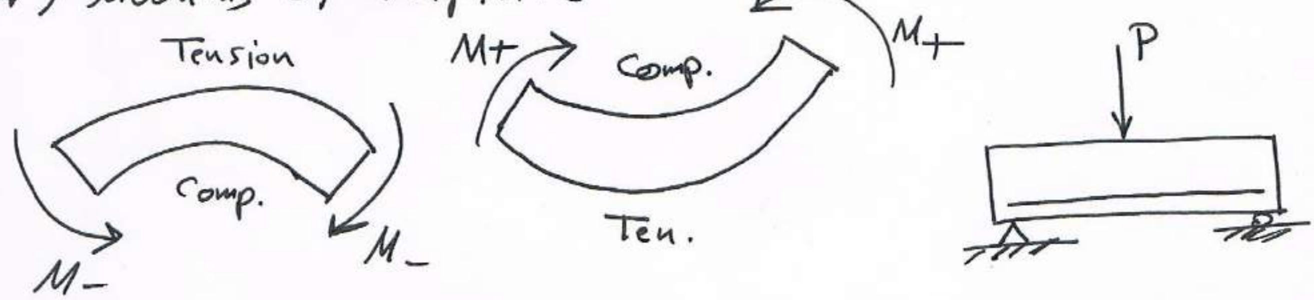
Behaviour of Reinforced Concrete Beam

There are 3 stages can be noticed when RC beam loaded until failure :-

1- Elastic - Uncracked Section (مرحلة ما قبل التشقق)

- الأحمال قليلة
- التناسب خطياً بين كل من الإجهادات والانفعالات
- لغرض تحليل المقطع المركب يحول حديد التسليح إلى ما يكافئه من الخرسانة (Transformed Section)
- يمكن تطبيق مبادىء وقوانين الميكانيك الهندسي في هذه المرحلة .
- تستمر هذه المرحلة لحين وصول الإجهادات إلى معامل الكسر

(f_r) Modulus of Rupture

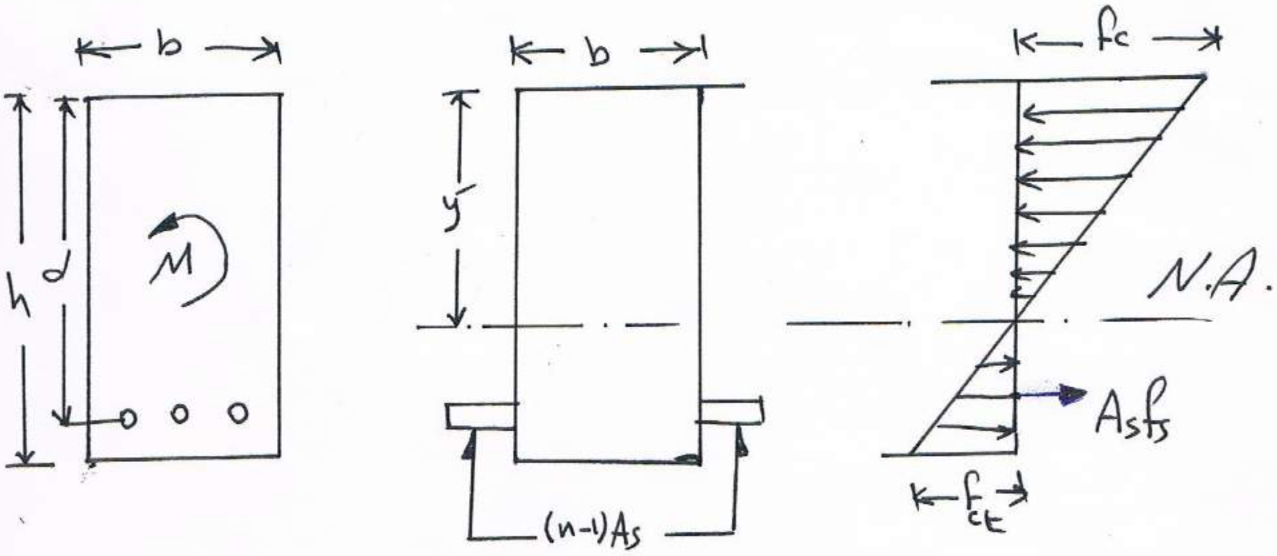


$f_c < 0.5 f_c'$ ---- Concrete is Elastic

$f_s < f_y$ .... Steel is Elastic

$f_{ct} < f_r$ Uncracked

$$n = \frac{E_s}{E_c} = \frac{200 * 10^3}{4700\sqrt{f_c'}}$$



Transformed Uncracked Section

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{(bh)(h/2) + (n-1)A_s * (d)}{bh + (n-1)A_s}$$

$$I_{N.A.} = \frac{b\bar{y}^3}{3} + \frac{b(h-\bar{y})^3}{3} + (n-1)A_s(d-\bar{y})^2$$

$$f_{ct} = \frac{M.C}{I_{N.A.}} = \frac{M_{max}(h-\bar{y})}{I_{N.A.}}$$ check $< f_r = 0.7\sqrt{f'_c}$

∴ Uncracked ok

$$f_c = \frac{M.C}{I_{N.A.}} = \frac{M_{max}(\bar{y})}{I_{N.A.}}$$ check $< f_c$ allowable $= 0.5 f'_c$

∴ ok Elastic

$$f_s = \frac{n.M(d-\bar{y})}{I_{N.A.}}$$ check $< f_y$ ∴ ok Elastic

2 - (Elastic - Cracked Section)

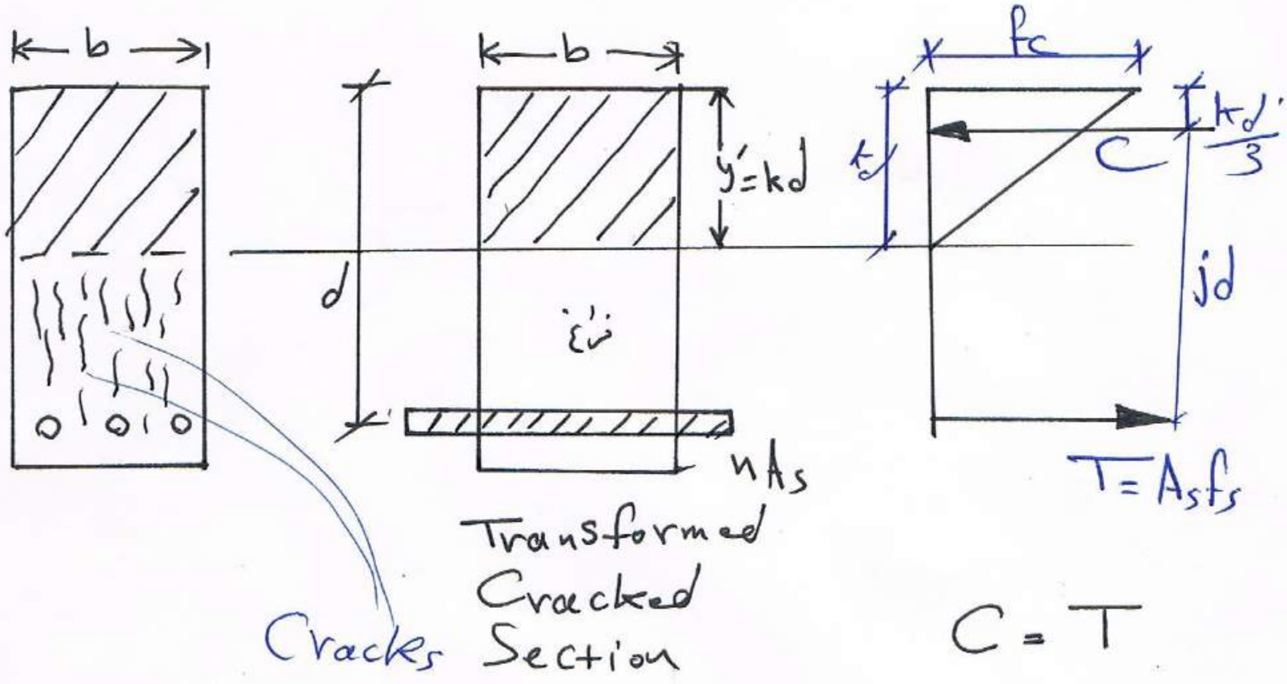
* مرحلة تشقق الخرسانة مع بقاء الاجهادات خطية

Load is increased

$f_{ct} > f_r$ ∴ Cracked section

$f_c < 0.5 f'_c$ Concrete is Elastic

$f_s < f_y$ ∴ Steel is Elastic



$C = T$

$M = T * jd$
$= C * jd$

$$n = \frac{E_s}{E_c} = \frac{200\,000}{4700\sqrt{f'_c}}$$

Find N.A. Position

عزم مساحة الشد حول N.A. = عزم مساحة الضغط حول N.A.

$$b \cdot kd \cdot \left(\frac{kd}{2}\right) = nA_s\,(d - kd) \quad \text{—— (1)}$$

Find N.A Position

Area of Compression about N.A = Area of Tension about N.A

$$b \cdot kd \cdot \left(\frac{kd}{2}\right) = n A_s (d - kd) \quad \text{---} (2)$$

$$\text{Steel Ratio} = \frac{A_s}{bd} \Longrightarrow A_s = \rho b d$$

Sub. in eq.(2)

$$b \cdot kd \cdot \left(\frac{kd}{2}\right) = n \rho b d (d - kd)$$

$$\frac{k^2}{2} = n\rho(1-k)$$

$$k^2 = 2n\rho - 2k\rho n$$

$$k^2 + 2\rho n k - 2\rho n = 0 \quad \text{---} (3)$$ تحل بالدستور

$$k = \sqrt{(\rho n)^2 + 2(\rho n)} - \rho n \quad \text{---} (4)$$

$$I_{N.A} = \frac{b(kd)^3}{3} + n A_s (d - kd)^2$$

$$f_c = \frac{MC}{I_{N.A}} = \frac{M_{max}\, kd}{I_{N.A}} < 0.5 f'_c \quad \therefore (\text{Elastic})$$

$$f_s = n \frac{MC}{I_{N.A}} = \frac{n M_{max}(d - kd)}{I_{N.A}} < f_y \quad \therefore (\text{Elastic})$$

Allowable Stresses of Materials according to ACI-Code

- Concrete $f_c = 0.45 f'_c$
- Steel Reinforcement $f_y = 300\,MPa \Rightarrow f_s = 140\,MPa$

  $f_y = 400\,MPa \Rightarrow f_s = 170\,MPa$

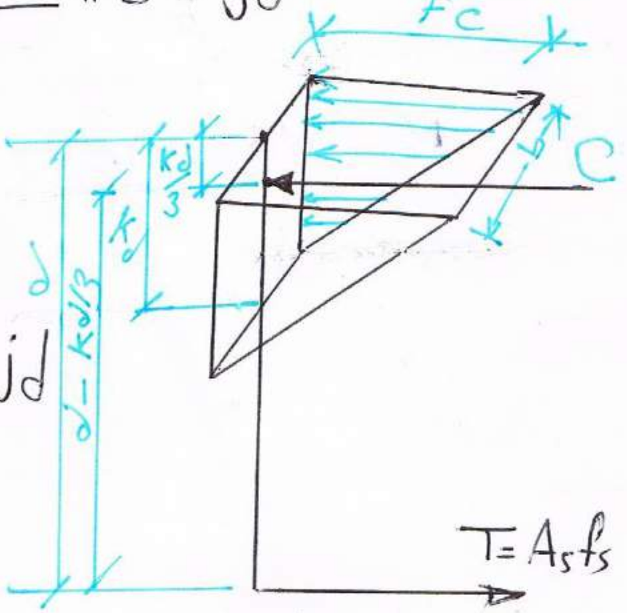
Method of Internal Moment أسلوب العزم الداخلي

$$M = C \cdot jd = \frac{f_c \cdot kd}{2} * b * jd$$

$$f_c = \frac{2M}{kjbd^2}$$

$$M = T \cdot jd = A_s f_s \cdot jd$$

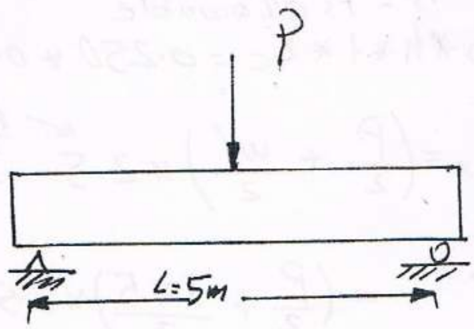
$$\therefore f_s = \frac{M}{A_s j d}$$



$$d = \frac{kd}{3} + jd \Longrightarrow$$ (للمقطع المستطيل فقط)

$$j = 1 - \frac{k}{3}$$

Ex:- Find Maximum Load (P) Can be applied at the Center of the beam shown below for these information:-

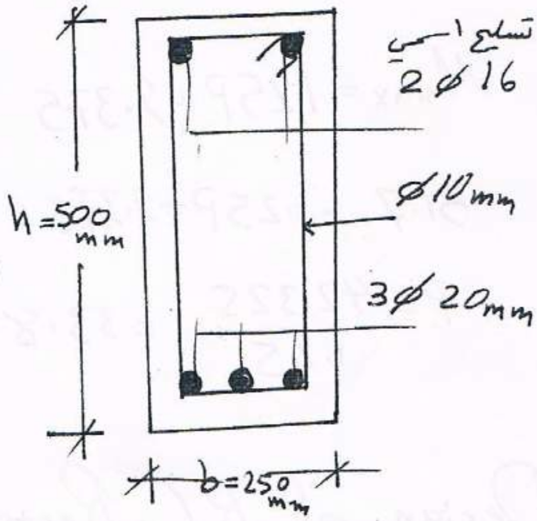


$b = 250\ mm$, $h = 500\ mm$

$A_s = 3\phi\ 20mm$, $E_s = 200\ 000\ N/mm^2$

$E_c = 22\ 000\ N/mm^2$, $\gamma_c = 24\ kN/m^3$

$f_y = 300\ MPa$, $f'_c = 20\ MPa$



Solution:- $d = 500 - (40 + 10 + \frac{20}{2}) = 440\ mm$

$A_s = 3 * \frac{\pi}{4}(20)^2 = 942\ mm^2$

$\rho = \frac{A_s}{bd} = \frac{942}{250 * 440} = 0.0088$

$n = \frac{E_s}{E_c} = \frac{200\ 000}{22\ 000} \approx 9.09$

$\rho n = 0.0088 * 9.09 = 0.08$

$K = \sqrt{(0.08)^2 + 2(0.08)} - (0.08) = 0.328$

$j = 1 - \frac{K}{3} = 0.891$

$jd = d - \frac{kd}{3} = 440 - \frac{0.328 * 440}{3} = 391.9\ mm$

$f_c = \frac{2M}{Kjbd^2}$, $f_{c_{all}} = 0.45 * 20 = 9.0\ MPa$

$M = 0.5 * f_{c_{all}} Kjbd^2 = 0.5 * 9 * 0.328 * 0.891 * 250 * (440)^2$
$= 63.652 * 10^6\ N.mm$

$f_s = \frac{M}{A_s jd}$, $f_y = 300\ MPa$, $f_{s_{all}} = 140\ MPa$

$M = f_{s_{all}} A_s j d = 140 * 942 * 0.891 * 440 = 51.7 * 10^6\ N.mm$
$= 51.7\ kN.m$

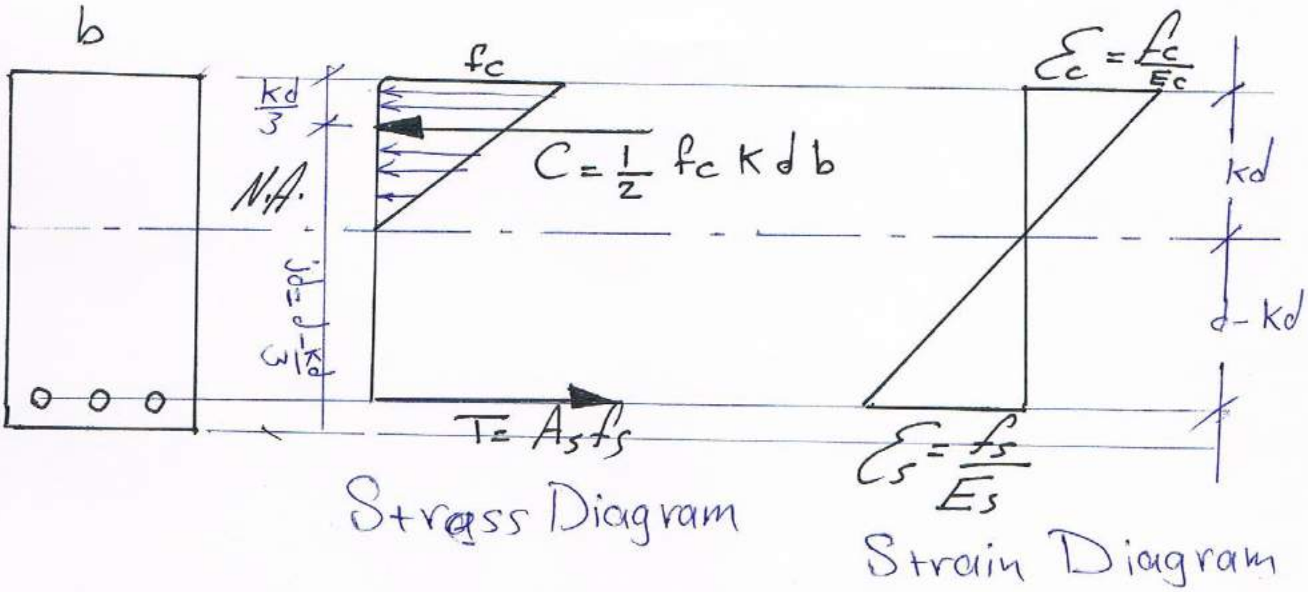
M allowable is the smaller of the moments that make $f_c = f_{c_{all}}$

Design of R.C. Rectangular Beams by W.D. Method

- Design of R.C. Beam means finding the dimensions of section (b x h) and the details and area of steel reinforcement (A_s), Number of bars and their distribution to withstand internal stresses which were caused by external loads.

Design consider to be done by both concrete and steel reinforcement reach the allowable stresses in the same time. In this case, the section is called balanced Section.

Balanced section is economical section because it is used both of steel and concrete properties in high level.



From strain diagram

$$\frac{\frac{f_c}{E_c}}{kd} = \frac{\frac{f_s}{E_s}}{d-kd} \Longrightarrow \frac{\frac{E_s}{E_c}}{k} = \frac{\frac{f_s}{f_c}}{1-k}$$

Let $r = \frac{f_{s\,all}}{f_{c\,all}}$, $\frac{n}{k} = \frac{\frac{f_s}{f_c}}{1-k}$

$\frac{f_s}{f_c} = \frac{n(1-k)}{k}$, in balance conditions $r = \frac{n(1-k_b)}{k_b}$

$$r k_b = n - n k_b \Longrightarrow r k_b + n k_b = n$$

$$k_b(r+n) = n \Longrightarrow \boxed{k_b = \frac{n}{n+r}}$$

From Stress Diagram

$$T = C \Longrightarrow A_s f_s = \frac{1}{2} f_c k d b$$

$$\frac{A_s}{bd} * \frac{f_s}{f_c} = \frac{1}{2} * k \Longrightarrow \rho \frac{f_s}{f_c} = \frac{k}{2}$$

in balance conditions $\rho_b \frac{f_{s\,all}}{f_{c\,all}} = \frac{k_b}{2} \Rightarrow \boxed{\rho_b = \frac{k_b}{2r}}$

$$\boxed{\rho_{min} = \frac{1.4}{f_y}} \quad \text{according to ACI-code}$$

$$\rho_b > \rho > \rho_{min}$$

E.X.

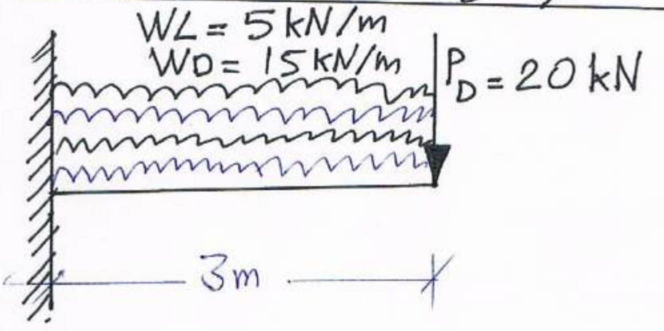


Fig.

Design the cantilever shown in the Fig. below using the following data. $f'_c = 20 N/mm^2$, $f_y = 275 N/mm^2$; $E_s = 200\,000 N/mm^2$, $\gamma_c = 24 kN/m^3$

Solution:-

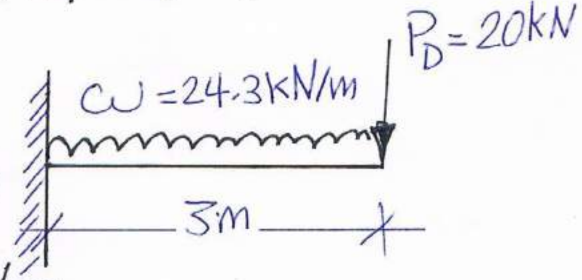
- Assume depth of cantilever $= \frac{L}{5} = h$
- Assume width of cantilever $(b) = \frac{h}{2} = \frac{L}{5}/2 = \frac{L}{10}$

$$\omega_{self} = b \times h \times 1 \times \gamma = \frac{h}{2} \times h \times 24$$

$$= \frac{L}{5} \times \frac{L}{10} \times 24 = 4.32 \text{ kN/m}$$

$$\omega_{total} = \omega_L + \omega_D + \omega_{self}$$

$$= 5 + 15 + 4.3 = 24.3 \text{ kN/m}$$



$$M_{max} = P_D \cdot L + \frac{\omega L^2}{2} = 169.44 \text{ kN.m}$$

$$\rho_b = \frac{k_b}{2r} \Rightarrow \quad k_b = \frac{n}{n+r}, \quad r = \frac{f_{s\,all}}{f_{c\,all}} = \frac{140}{0.45 * 20} = 15.55$$

$$n = \frac{200\,000}{4700\sqrt{20}} = 9.52$$

$$k_b = \frac{9.52}{9.52 + 15.55} \simeq 0.38, \quad j = 1 - \frac{k}{3} = 1 - \frac{0.38}{3} = 0.8733$$

$$\rho_b = \frac{0.38}{2 * 15.55} = 0.0122 \quad , \quad \rho_{min} = \frac{1.4}{275} = 0.005$$

Use $\rho = 0.01$

$$M = M_s = \rho f_s j b d^2 \Longrightarrow 169.44 * 10^6 = 0.01 * 140 * 0.8733 * b * (2b)^2$$

$$4b^3 = 138.587 * 10^6$$

$$b = \sqrt[3]{34.64 \times 10^6} = 326 \text{ mm} \qquad \text{USE } b = 330 \text{mm}$$

$$d = 2b = 2 * 330 = 660 \text{mm}$$

$$A_s = \rho b d = 0.01 * 330 * 660 = 2178 \text{mm}^2$$

Use ϕ 22mm , $A_b = \frac{\pi}{4} * 22^2 = 380 \text{ mm}^2$

$$\text{No of bars} = \frac{2178}{380} = 5.73$$

Use 6ϕ22

- The distanse between each bar must not be less than 25mm
- The concrete cover from each sides must not be less than 100mm (i.e. 50mm for each side)
- If we put all bars in the same layer ∴ the width of the beam will be equal to

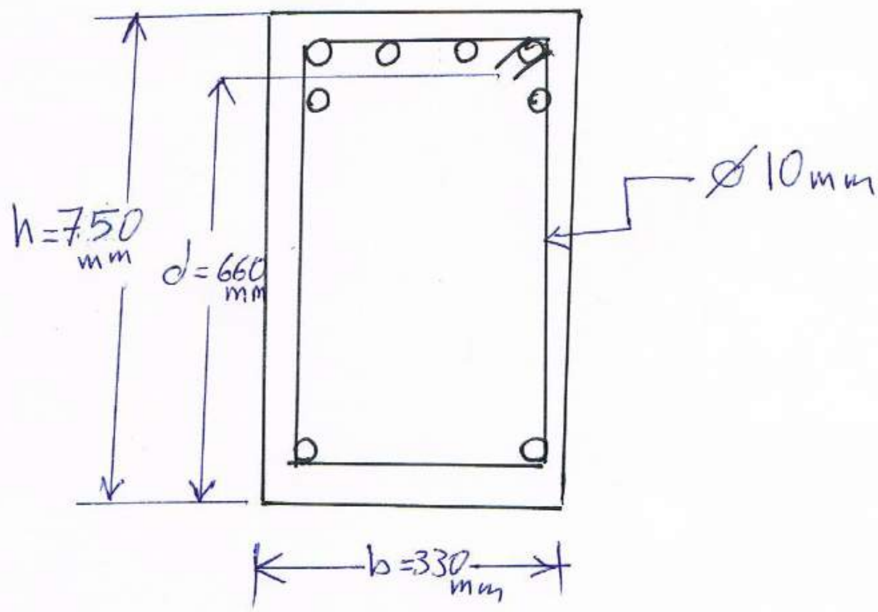
$$6 \times 22 + 5 \times 25 + 100 = 357 \text{mm} > b = 330 \text{ mm}$$

- ∴ We distribute the bars into (2) layer one of them contains 4ϕ22 & the other contains 2ϕ22
- $4 \times 22 + 3 \times 25 + 100 = 263 \text{mm} < b = 330 \text{mm}$ ∴ ok

$$h = d + \frac{\text{the distance between 2 bars}}{2} + d_{bar} + d_{stirup} + \text{Cover}$$

$$h = 660 + \frac{25}{2} + 22 + 10 + 40$$

$$= 744.5_{mm} \Longrightarrow \text{USE } h = 750\,mm$$



Ultimate Strength Design Method (S.D.M)

The assumptions which are used in this method:-

1- Internal force = external force
2- Cross section before bending remain plain after bending
3- Concrete capable of no tensile stress.
4- Theory is based on actual stress-strain relationship.
5- The strain in steel = strain in surrounding concrete.
6- There is complete bond between concrete and steel.
7- Maximum stress in steel must not exceed (f_y).

Safety Factors: S.F.

S.F. can be defined in 2 ways:-

a- $S.F. = \frac{Max.\ Stress}{Allowable\ Stress}$ (W.S.D.M)

b- $S.F. = \frac{Max\ Load}{Servise\ Load}$ (S.D.M)

Load Factors

* $U = 1.2D + 1.6L$ —— (1)
* $U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.8W)$ — (2)

W- wind load R- rain load D- dead load
S- snow load Lr- roof load (Live load) L- live load

* $U = 1.2D + 1.6W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$ — (3)
* $U = 0.9D + 1.6W$ —— (4)

When tension forces which be occured according to wind on the members, then the vertical forces can be decreased to the mimum value, i.e. L.L = Zero - DL will decreased by 10%.

The load factors in the equations above represent the mimum values and the desinger can increase thes factors according to his opinion.

Strength Reduction Factors

- Tension $\phi = 0.9 \Rightarrow M_u = \phi M_n$

  M_u = Ultimate moment capacity
  M_n = nominal (actual) moment capacity

- Shear, Torsion $\phi = 0.75 \Rightarrow V_u = \phi V_n$

  V_u = ultimate shear capacity
  V_n = nominal shear capacity

- Compression

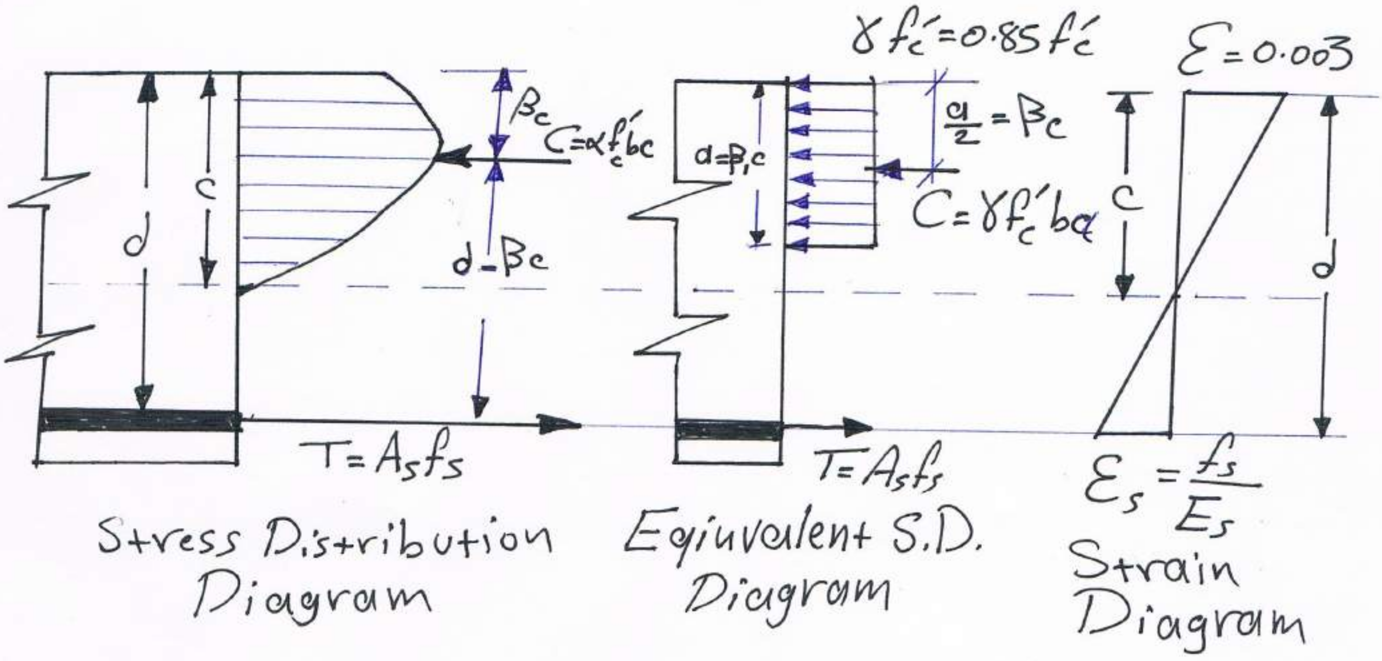
  a- $\phi = 0.70$, for spiral reinforced members like columns.
  b- $\phi = 0.65$, for other reinforced members like columns.

Stress and Strain Distribution

Resultant of Concrete Comprssive Forces:-

$$C = f_{av}\, b\, c$$

where f_{av}: average compressive stress in concrete
b: the width of section
c: the depth of Nutral Axis.



$$C = \alpha f'_c b c$$

were $\alpha = \frac{\text{average concrete stress}}{\text{concrete compressive strength}}$

- The location of the resultant is usually represented by βc were $\beta = \frac{\text{compressive resultant depth}}{\text{N.A. depth}}$

$\alpha = 0.72$ for $f'_c \leq 30 MPa$

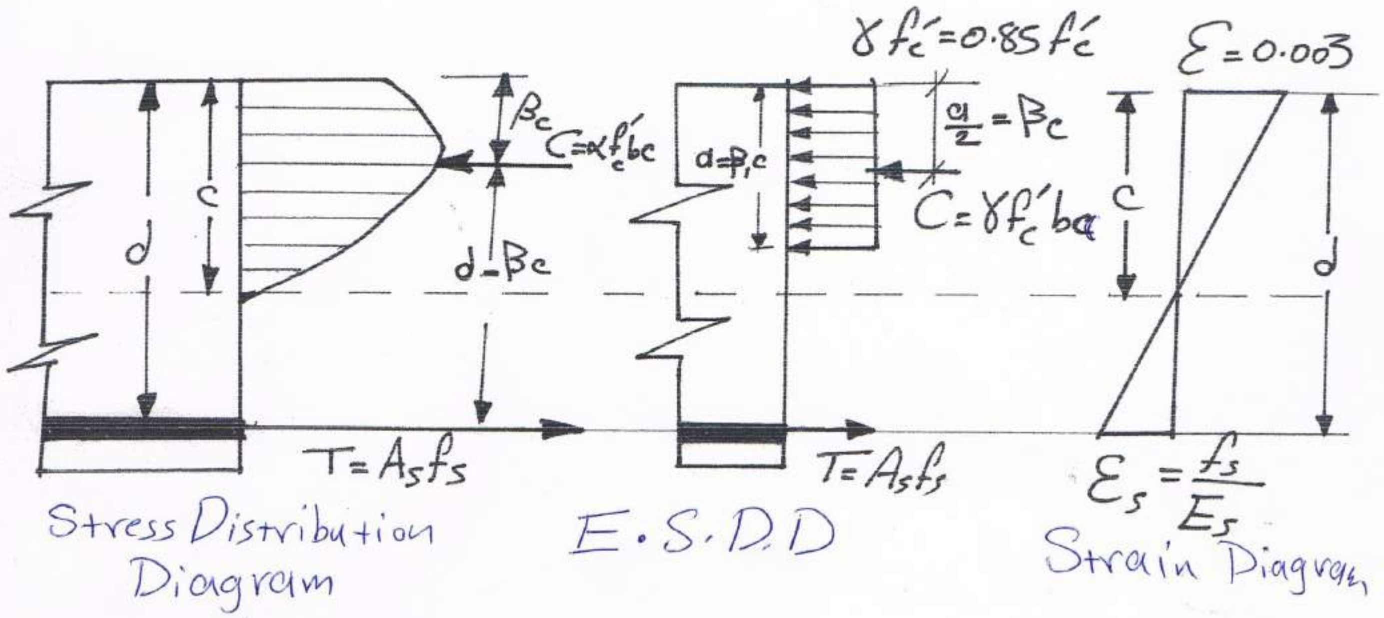
α decreased by (0.04) for every (7MPa) increasing in compressive strength of concrete.

α value must not be less than ($\alpha = 0.56$).

$\beta = 0.425$ for $f'_c \leq 30 MPa$

β decreased by (0.025) for every (7 MPa) increasing in compressive strength of concrete.

β value must not be less than ($\beta = 0.325$)



Equivelant rectangular stress block is used for analysis of reinforced concrete sections.

$$C = \alpha f'_c c b = \gamma f'_c a b \quad \text{—— (1)}$$

Let $a = \beta_1 c$ —— (2)

We can find γ, β_1, α, β

$$\frac{a}{2} = \beta c \qquad \therefore a = 2\beta c$$

From eq (2) $\quad \beta_1 c = 2\beta c \quad \therefore \beta_1 = 2\beta$ —— (3)

Sub in eq. (1) :-

$$\alpha f'_c c b = \gamma f'_c a b$$

$$\gamma = \frac{\alpha c}{a} \Rightarrow \gamma = \frac{\alpha c}{2\beta c} \Rightarrow \gamma = \frac{\alpha}{2\left(\frac{\beta_1}{2}\right)}$$

$$\gamma = \frac{\alpha}{\beta_1} \quad \text{—— (4)}$$

from the above equations & from the value of (β, α) we can find the values of $(\beta_1$ & $\gamma)$:- $\beta_1 = 2\beta = 2 * 0.425 = 0.85$

$$\gamma = \frac{\alpha = 0.72}{\beta_1 = 0.85} \simeq 0.85 \quad \text{—— (5)}$$

$\beta_1 = 0.85$ for $f'_c \leq 30$ MPa

β_1 decreased by (0.05) for every (7MPa) increasing in compressive strength of concrete

β_1 value must not be less than ($\beta_1 = 0.65$)

$$\beta_1 = 0.85 - 0.05(f'_c - 30)/7$$

Analysis and Design of Singly Reinforced Rectangular Beams

a- Balanced or Under Reinforced Beams

$$\rho \leq \rho_b \Rightarrow f_s = f_y$$

from the equilebrium conditions

$$C = T$$

$$0.85 f'_c * b * a = A_s f_y$$

$$a = \frac{A_s f_y}{0.85 f'_c b} \quad \text{—(a)}, \quad A_s = \rho b d \quad \text{—(b)}$$

$$a = \frac{\rho b d \, f_y}{0.85 f'_c b} \Rightarrow a = \frac{\rho f_y d}{0.85 f'_c} \quad \text{—(1)}$$

$$M_n = A_s f_y * \left(d - \frac{a}{2}\right) \quad \text{—(2)}$$

$$M_n = 0.85 f'_c \cdot a \cdot b \left(d - \frac{a}{2}\right) \quad \text{—(3)}$$

sub a & b in equ (2)

$$M_n = \rho b d f_y \left[d - \frac{\rho f_y d}{2 \; 0.85 f'_c}\right]$$

$$M_n = \rho b d^2 f_y \left[1 - \frac{0.59 \rho f_y}{f'_c}\right]$$

$M_u = \phi M_n$

$$M_u = \phi \rho b d^2 f_y \left[1 - \frac{0.59 \rho f_y}{f'_c}\right] \quad \text{—— (4)}$$

b- <u>Over Reinforced Beams</u> $\therefore \rho > \rho_b$

f_s = unknown $> f_y$

$$A_s f_s = 0.85 f'_c \cdot a \cdot b$$

$$A_s f_s = 0.85 f'_c \cdot (\beta_1 c) \cdot b$$

There are (2) unknowns f_s and c

After many steps:-

$$m = \frac{600}{0.85 \beta_1 f'_c} \quad , \quad k_u = \sqrt{\frac{(\rho m)^2}{2} + \rho m} - \frac{\rho m}{2}$$

Then we can find the nominal strength by the following procedure:-

1- find ρ, m were $\rho = \frac{A_s}{bd}$, $m = \frac{600}{0.85 \beta_1 f'_c}$

2- submit in $k_u = \sqrt{\frac{(\rho m)^2}{2} + \rho m} - \frac{\rho m}{2}$

3- calculate c value were $c = k_u \cdot d$

4- calculate a value were $a = \beta_1 c$

5- find f_s were $f_s = 600 - \frac{(d-c)}{c}$

6- find the nominal bending moment M_n by using on of the three equations (2), (3) and (4)

Design by Ultimate Design Method:-

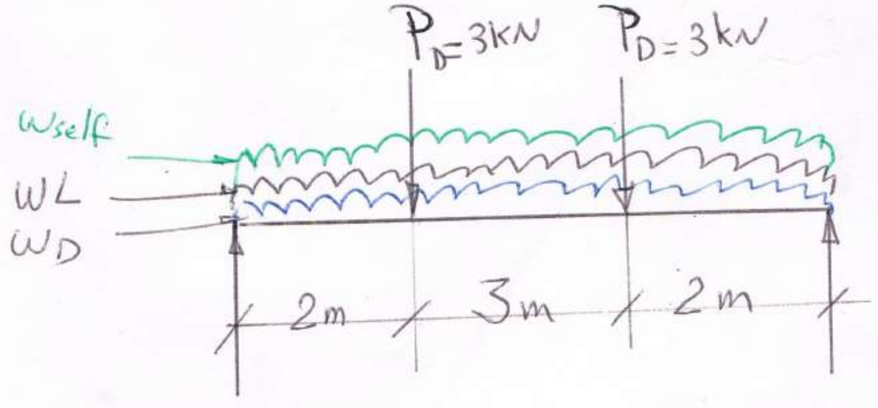
* The design of R.C. Members means finding the adiquate dimensions for these members & the reinforcement magnitude to enable the member to withstand maximum loads applied on it safely.

* Sometimes all dimensions or some of them are determined by archetectures.

* Complete design for the beam requires determine the shear reinforcement, torsion reinforcement & check defelections, check development lengths and the points of cut or bend of steel reinforcement. All these details must be put on the beam scketh or diagram.

Ex :- Design the beam shown below for the following datas:- $f'_c = 20 N/mm^2$, $f_y = 300 N/mm^2$

$\gamma'_c = 24 kN/m^3$, $W_L = 6 kN/m$, $W_D = 12 kN/m$



Solution :-

1* Find the $M_{u\,max}$.

* for cantilever, assume $h = \frac{L}{5}$
  for simply supported beam & continous beam, assume $h = \frac{L}{10}$

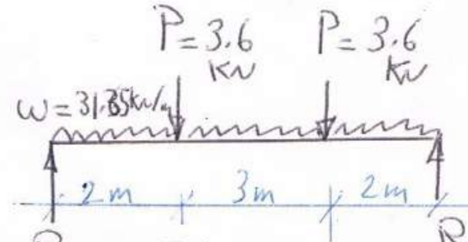
* $b = \frac{h}{2}$, $\therefore b = \frac{L}{10}/2 = \frac{L}{20}$

$$\omega_{self} = b*h*1*\gamma_c = \frac{L}{20}*\frac{L}{10}*1*24 = \frac{L^2}{8.33} \simeq \frac{L^2}{8} (kN/m)$$

$$\omega_{total} = \omega_{self} + \omega_D + \omega_L$$

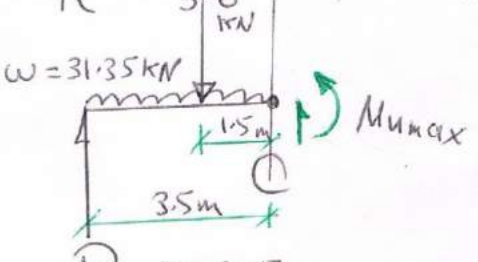
$$= 1.2\left(\frac{(7)^2}{8} + 12\right) + 1.6(6) = 31.35 \; kN/m$$

$$P = 1.2 P_D = 1.2 * 3 = 3.6 \; kN$$



$\sum F_y = 0$

$$R = \frac{3.6*2 + 31.35*7}{2} = 113.325 \; kN$$



$\sum M_{(c)} = 0$

$$M_{umax} = R*3.5 - P*1.5 - \frac{\omega(3.5)^2}{2}$$

$$= 113.325*3.5 - 3.6*1.5 - \frac{31.35(3.5)^2}{2}$$

$$M_{umax} = 199.219 \; kN \cdot m$$

2- ρ_{min}, ρ_{max} & ρ_t

* from Table(3), $\rho_{min} = 0.0047$

Or $\rho_{min} = \frac{1.4}{f_y} = \frac{1.4}{300} = 0.00467$ ✓

$\rho_{min} = \frac{\sqrt{f'_c}}{4 f_y} = \frac{\sqrt{20}}{4*300} = 0.00373$

use the bigst value

* ρ_{max} :- from Table(3):- $\rho_{max} = 0.0206$

or by the eq. :- $\rho_{max} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{0.003}{0.003 + \varepsilon_t}$ $(\varepsilon_t = 0.004)$

$(\beta_1 = 0.85 \text{ for } f'_c \leq 30 MPa)$ $\rho_{max} = 0.85 * (0.85) * \frac{20}{300} * \frac{0.003}{0.003 + 0.004}$

$\rho_{max} = 0.02064$

• We must use $\rho \Longrightarrow \rho_{min} < \rho \leq \rho_{max}$

3- • for use $\phi = 0.9$ ρ must be $\leq \rho_t$

$\rho_t = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{0.003}{0.003 + \varepsilon_t}$ $(\varepsilon_t = 0.005)$

$\rho_t = 0.85 (0.85) * \frac{20}{300} * \frac{0.003}{0.003 + 0.005} = 0.01806$

or from Table(3) $\rho_t = 0.0180$

∴ USE $\rho = 0.0170$

4- $M_u = \phi \rho b d^2 f_y \left(1 - 0.59 \rho \frac{f_y}{f'_c}\right)$

$199.22 * 10^6 = 0.9 * 0.0170 * bd^2 * 300 \left(1 - 0.59 * 0.017 * \frac{300}{20}\right)$

$199.22 * 10^6 = 4.59 bd^2 - 0.6906 bd^2$

$bd^2 = 51089459.3$

assume $b = \frac{d}{2}$

$\frac{d}{2} * d^2 = 51089459.3$

$d^3 = 102.179 * 10^6$

$d = \sqrt[3]{102.179 * 10^6} = 467.5 \text{ mm} \Rightarrow$ USE $d = 470$ mm

5- $A_s = \rho b d = 0.0170 * 240 * 470 = 1917.6 \simeq 1918 mm^2$

Use $\phi_{bar} = 22 mm$ $\quad A_{s\,bar} = \frac{\pi}{4} * (22)^2 = 380.13 mm^2$

(or from Table (1):-

$N^{\underline{o}}$ of bars $= \frac{A_s}{A_{s\,bar}} = \frac{1918}{380} = 5.047 \simeq 5$

6- • $S \geqslant \begin{cases} 25 mm \\ \phi_{bar} \\ \frac{4}{3} * \text{max. Size of aggregte} \end{cases}$ distance between bars

• Thickness of the cover:-

a- cover $\geqslant$ 75mm (on the ground)

b- cover $\geqslant$ 25mm (concrete in contact with soil or with invironment conditions)
(for Slabs & عتبات & wall slabs)

* for other concrete members = 40 mm

c- Cover $\geqslant$ 20mm (concret is not in contact with soil or other conditions)
(slabs, عتبات & walls)

* for beams & columns = 40mm & for secondary = 25mm

$S = (b - 2(\overset{40}{cover} + \phi_{s\,bar}) - n\phi_{bar}) / (n-1)$

n = $N^{\underline{o}}$ of bars

b = Width of the section

ϕ_{bar} = bar diameter

ϕ_s or $\phi_{s\,bar}$ = diameter of shear reinf.

assume the steel bars are distrebuted in one layer.

$S_{actuale} = [240 - 2*(40+10) - 5*22] / (5-1) = 7.5 mm$

$S = \begin{cases} 25 mm \checkmark \\ \phi_b = 22 mm \\ \frac{4}{3} * \text{max. of agg.} \end{cases}$ $\therefore S = 25 mm$

$\because S_{act} < S_{min} = 25 mm$

$\therefore$ let us use the reinf. in two layers

5 bars < 3 bars, 2 bars

$S_{act} = [240 - 2(40+10) - 3*22] / (3-1) = 37 mm > 25 mm$

$\therefore$ O.K.

$h = d + 70 mm$ (one layer)

$h = d + 100 mm$ (two layers)

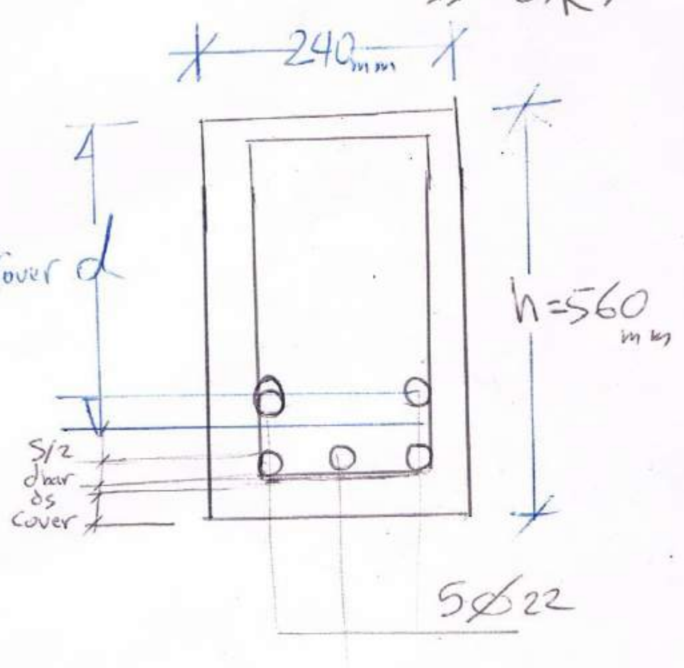
$h = d + 130 mm$ (3 layers)

or actual $h = d + \frac{S}{2} + d_{bar} + d_s + \text{cover}$

$h = 470 + 25/2 + 22 + 10 + 40$

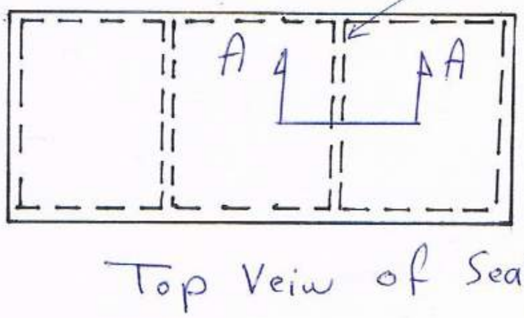
$= 554.5 mm$

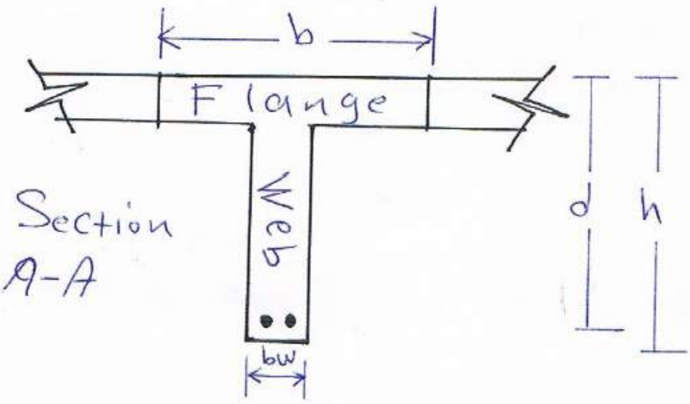
Use $h = 560 mm$



Analysis and Design of T-Beams

Because slabs and beams are ordinarily cast together, the beams are automatically provided with an extra width at the top that called a (Flange). Such beams are known as T-beams. The portion below the flange is called the (Web).

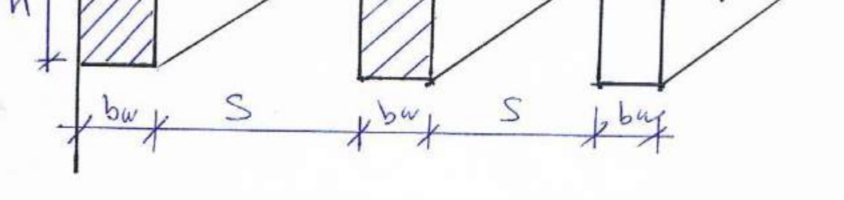




ACI-Code (ACI-8.10) prescribes a limit on the effective flange width (b) of interior T-sections to the smallest of the following :-

1- $b \leq \frac{L}{4}$
2- $b \leq 16h_f + b_w$
3- $b \leq S + b_w$

The Smallest Value



For the exterior (L-Section)

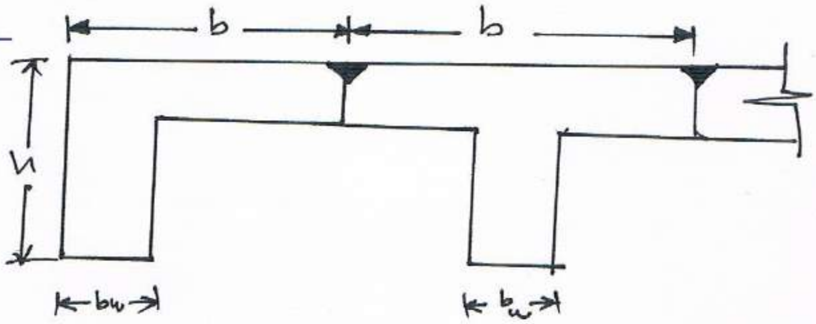
1- $b \leq b_w + \frac{L}{12}$
2- $b \leq b_w + 6h_f$
3- $b \leq b_w + S/2$

The Smallest Value

Isolated T- Sections

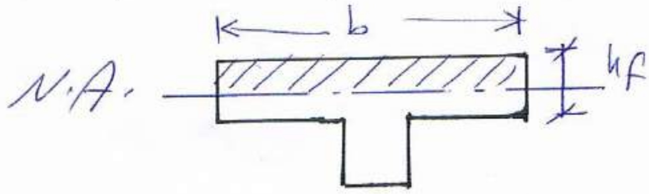
$h_f \geq b_w/2$

$b \leq 4b_w$

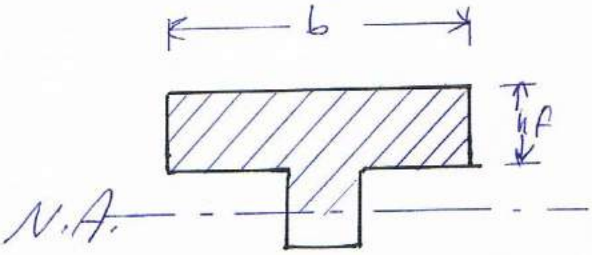


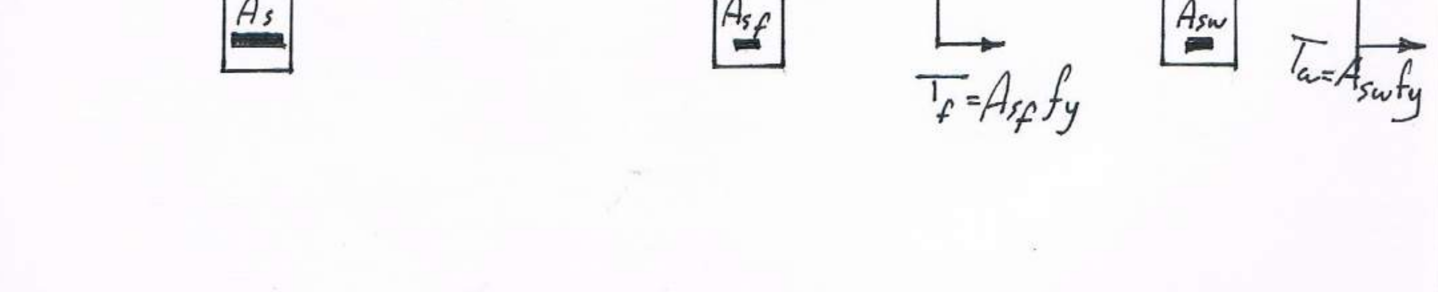
Analysis of T-beams

(1) If the calculated depth to the neutral axis less than or equal to slab thickness (h_f), then the beam can be analyzed as rectangular beam of width equal to (b)



(2) If the calculated depth to the neutral axis is greater than the slab thickness (h_f), then the beam will behaive as T-beam.





Analysis of T-beam

1- Find the depth of compressive area

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

2- If $a \leqslant h_f$ $\xrightarrow{\text{then}}$ the analysis will be as rectangular beam with (width = b) and (depth = d).

3- If $a > h_f$ $\xrightarrow{\text{then}}$ $A_{sf} = \frac{0.85 f'_c (b - b_w) h_f}{f_y}$

A_{sf}: Area of steel required to equilified the compressive stress of flange

Find ρ_w , $\rho_w = \frac{A_s}{b_w d}$

Find ρ_{wb} , $\rho_{wb} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{600}{600 + f_y} + \rho_f$

$$\rho_{wb} = \rho_b + \rho_f$$

4- If $\rho_w \leqslant \rho_{wb} \Longrightarrow a = \frac{A_{sw} f_y}{0.85 f'_c b_w}$

and find M_n from one of the two eqs

$$M_n = M_{n1} + M_{n2} = A_{sf} f_y \left(d - \frac{h_f}{2}\right) + A_{sw} f_y \left(d - \frac{a}{2}\right)$$

or

$$M_n = M_{n1} + M_{n2} = 0.85 f'_c \left[(b - b_w) h_f * \left(d - \frac{h_f}{2}\right) + a \cdot b_w \cdot \left(d - \frac{a}{2}\right)\right]$$

5- If $\rho_w > \rho_{wb}$ then we must calculate (c) by this equation

$$A_s \cdot 600 \cdot \left(\frac{d-c}{c}\right) = 0.85 f'_c (b-b_w) h_f + 0.85 \beta_1 f'_c \, c \, b_w$$

$$a = \beta_1 / c$$

then find M_n :-

$$M_n = 0.85 f'_c (b-b_w) h_f \cdot \left(d - \frac{h_f}{2}\right) + 0.85 f'_c \, a \, b_w \left(d - \frac{a}{2}\right)$$

6-

If $\varepsilon_t = 0.005 \xrightarrow{\text{then}} \phi = 0.9$

$$\rho_{wt} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.005} + \rho_f = \rho_t + \rho_f$$

ρ_{wt} : Reinforcement ratio caused strain = (0.005) for T-beam

ρ_t : Reinforcement ratio cause strain = (0.005) for rectangular portion

$$\rho_f = \frac{A_{sf}}{b_w d} = \frac{3631}{360 \times 600} = 0.0168$$

$$\rho_b = (0.85)^2 \frac{f'_c}{f_y} \frac{600}{600 + f_y} = (0.85)^2 * \frac{20.7}{345} * \frac{600}{600+345} = 0.0289$$

$$\rho_{wb} = \rho_b + \rho_f = 0.0289 + 0.0168 = 0.0457$$

$$\rho_w = \frac{A_s}{b_w d} = \frac{6432}{360 * 600} = 0.0298 < \rho_{wb} = 0.0457$$

∴ The beam is under reinforced

$$A_{sw} = A_s - A_{sf} = 6432 - 3631 = 2801 \text{ mm}^2$$

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 \times f'_c \times b_w} = \frac{2801 * 345}{0.85 \times 20.7 \times 360} = 152.56 \text{ mm}$$

$$M_u = \phi M_n$$

$$\rho_{wt} = \rho_t + \rho_f = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.005} + \rho_f = (0.85)^2 * \frac{20.7}{345} * \frac{0.003}{0.003 + 0.005} + 0.0168$$

∵ $\rho_{wt} > \rho$ ∴ $\phi = 0.9$

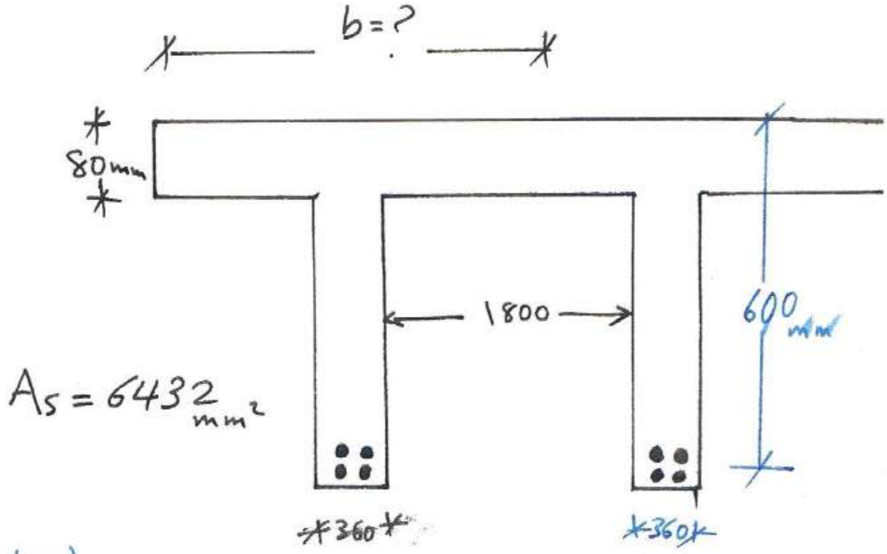
$$M_u = \phi \left[A_{sf} f_y \left(d - \frac{h_f}{2}\right) + A_{sw} f_y \left(d - \frac{a}{2}\right) \right]$$

$$M_u = 0.9 \left[3631 \times 345 \left(600 - \frac{80}{2}\right) + 2801 \times 345 \left(600 - \frac{152.56}{2}\right) \right]$$

$$M_u = 1086.8 * 10^6 \text{ N.mm} = 1086.8 \text{ kN.m}$$

Ex. :- An 80 mm thick continous slab is supported by rectangular beams as shown in the Fig. The span of the beam is 5m, $f'_c = 20.7$ MPa, $f_y = 345$ MPa, find the design strength of the T-beam

Solution :-



- $b = \frac{L}{4} = \frac{5000}{4} = 1250$ mm
- $b = 16 h_f + b_w$

$b = 80*16+360 = 1640$ mm

- $b = 360 + 1800$

$= 2160$ mm

Use $b = 1250$ mm (The smallest value)

- $a = \frac{A_s f_y}{0.85 f'_c b}$

- $a = \frac{A_s f_y}{0.85 * 20.7 * 1250} = \frac{6432 * 345}{0.85 \times 20.7 \times 1250} \Longrightarrow a = 100.9\,mm > 80\,mm$

∴ The beam is behaive as T-beam.

$$A_{sf} = \frac{0.85 f'_c (b - b_w) h_f}{f_y} = \frac{0.85 * 20.7 (1250 - 360) \times 80}{345} = 3631\,mm^2$$

Design of Doubly Reinforced Rectangular Beams

In design of singly reinforced beams (ρ) is be taken equal to (ρ_{max}) to insure tension failure. When the cross-section of beam is limited because of Architctur reasons or service reasons and its resistance strength is not enough to withstand Applied Moment. In this case, the solution is by adding compression steel instead of an equaivalent tensile steel to keep the Neutral Axis (N.A.) in the same position in the case of ($\rho = \rho_{max}$) to ensure tensile failure.

To calculate the steel reinforcement of both, tension and compression the next procedure must be do.

1- Calculate the design moment from structural analysis.
2- Find (ρ_{max}) from equation or table(P3).
3- Find (ρ) value from equation or table (P4), and if $\rho \leq \rho_{max}$ then the section is singly beam designed as singly reinforced beam, or the s is doubly and it will be designed as the next steps.
4- Find maximum design moment (Mu_{max}) which will be genrate by Maximum allowd steel reinforcement area (As_{max}) and here will be call it (A_{s1}), and we will call Mu_{max} (M_{u1}). for this case $\phi = 0.483 + 83.3\,\varepsilon_t = 0.816$

$$A_{s1} = \rho_{max} \cdot b \cdot d$$

We can use $\rho = \rho_t \Longrightarrow$ to ensure $\phi = 0.9$

$$a = \frac{A_{s1}\, f_y}{0.85\, f'_c\, b}$$

$$M_{u1} = \phi M_n = \phi A_{s1} f_y (d - \frac{a}{2})$$

5- Calculate design moment which withstand compression Steel (A'_s) and the equivelant tensile Steel reinforcement and the design moment must equal to :-

$$M_u = M_{u1} + M_{u2}$$

$$M_{u2} = M_u - M_{u1}$$

where :- M_u = design moment results from Str. analysis

M_{u1} = design moment results from tension reinforcement steel and concrete compressive

M_{u2} = design moment results from compression steel reinforcement and the equivelant tensile steel reinforcement.

6- Calculate compression steel stress.

$$C = a / B_1$$

$$\mathcal{E}'_s = \frac{c - d'}{c} \mathcal{E}_u$$

$$f'_s = E_s \mathcal{E}'_s = 600 \frac{c - d'}{c} \leq f_y$$

7- Calculate compression steel area from equilebrium eq.

$$A'_s = \frac{M_{u2}}{\phi f'_s (d - d')}$$

8- Calculate equivelant tensile steel area (balanced by comp. Steel)

$$A_{s2} f_y = A'_s f'_s \quad \therefore \quad A_{s2} = \frac{A'_s f'_s}{f_y}$$

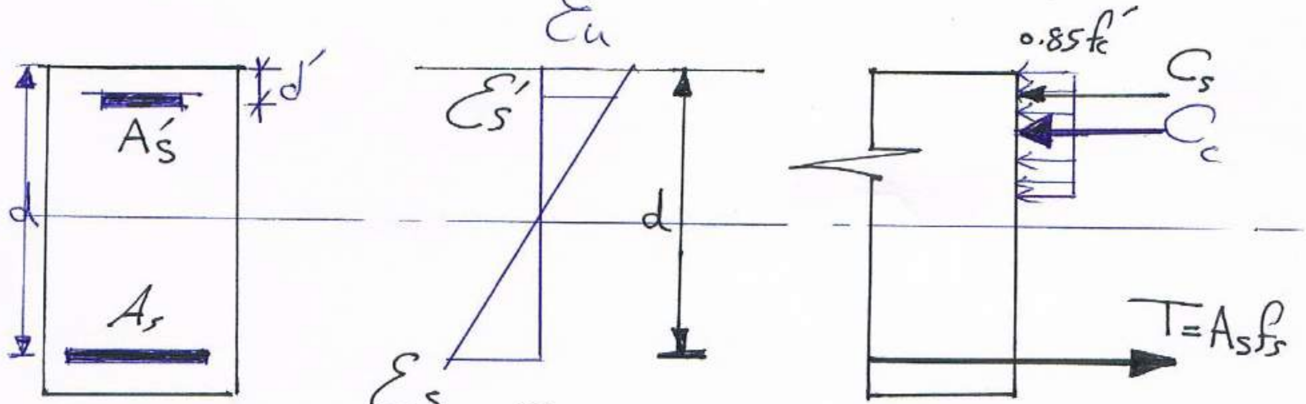
9- Find total area of tensile steel

$$A_s = A_{s1} + A_{s2}$$

10- Chose the diameter of steel reinforcement bar and find the number of these bars, then check the distances among bars according to ACI-Code requirements.

Analysis and Design of Doubly Reinforced Rectangular Beams

If a beam cross section is limited because of architectural or other considerations, it may happen that the concrete can not develop the compression force required to resist the given bending moment. In this case, reinforcement is added in the compression zone, resulting in a so-called doubly reinforced beam, i.e., one with compression as well as tension reinforcement.



Doubly Reinforced Cross-Section — Strain Diagram — Stress Distribution Diagram

A_s' : area of steel reinforcement for compression

$$\rho' = \frac{A_s'}{bd}$$

ρ'_{cy} : minimum tensile reinforcement ratio that will ensure yielding of the compression steel at failure.

$$\rho'_{cy} = 0.85\beta_1 \frac{600}{600-f_y} \cdot \frac{f_c'}{f_y} * \frac{d'}{d} + \rho'$$

if $\rho \geq \rho'_{cy} \Longrightarrow$ Compression Steel yield at failure.

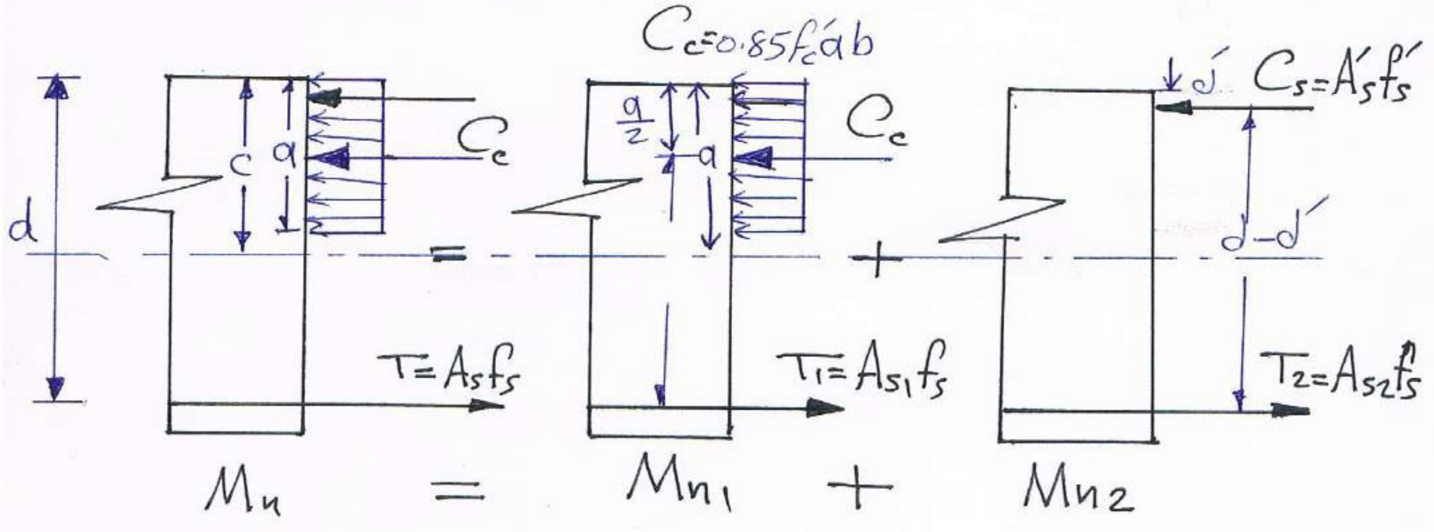
if $\rho < \rho'_{cy} \Longrightarrow$ Compression Steel Stress will not reach (f_y) at failure.

Compression reinforcement may be added according to another consideration like:-

- Decreasing the deflection resulting from creep.
- Fixing the shear reinforcement.
- Resistance of tensile force resulting from changing of moment.

Analysis of Doubly Reinforced Rectangular Beams:-

a- Tension and Compression Steel Both at Yield Stress:-



$$A'_s f_y = A_{s2} f_y \qquad \therefore A'_s = A_{s2}$$

$$A_{s1} = A_s - A_{s2} \qquad \therefore A_{s1} = A_s - A'_s$$

from force equilibrium:

$$A_{s1} f_y = 0.85 f'_c a b \implies a = \frac{A_{s1} f_y}{0.85 f'_c b}$$

$$a = \frac{(A_s - A'_s) f_y}{0.85 f'_c b}$$

$$M_n = M_{n1} + M_{n2} = A_{s1} f_y \left(d - \frac{a}{2}\right) + A_{s2} f_y (d - d')$$

$$M_n = M_{n1} + M_{n2} = 0.85 f'_c a b \left(d - \frac{a}{2}\right) + A'_s f_y (d - d')$$

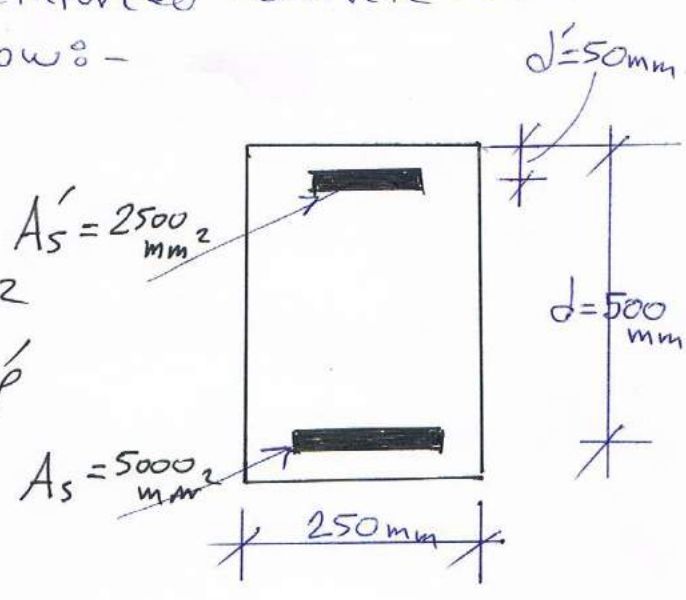
ρ_b' : balanced reinforcement ratio for doubly reinforced beam

$$\rho_b' = \rho_b + \rho'$$

ρ_b: balanced reinforcement ratio for corresponding Singly reinforced beam

$$\rho_{max}' = \rho_{max} + \rho'$$

Ex. :- Find the nominal moment for the cross section of doubly rectangular reinforced concrete beam shown in the figure below:-



$f_y = 350MPa$, $f_c' = 30MPa$

Solution

$\rho = \frac{5000}{250 \times 500} = 0.04$, $\rho' = \frac{2500}{500 \times 250} = 0.02$

$$\rho_{cy}' = 0.85\beta_1 \frac{f_c'}{f_y} \frac{d'}{d} \cdot \frac{600}{600-f_y} + \rho'$$

$$\rho_{cy}' = 0.0349$$

$$\rho_b = 0.85\beta_1 \frac{f_c'}{f_y} \cdot \frac{600}{600+f_y}$$

$\rho_b = 0.039$ $\quad \rho = 0.04 > \rho_{cy}' = 0.0349$ ∴ both tension & compression Steel at yield Stress

$\rho_b' = \rho_b + \rho' = 0.059$

$\rho = 0.04 < \rho_b'$, $a = \frac{(5000-2500) * 350}{0.85 * 30 * 250} = 137.254$ mm

$$\therefore M_n = 2500 \times 350 \left(500 - \frac{137.254}{2}\right) + 2500 \times 350 (500 - 50)$$

$M_n = 771.2 \times 10^6 \ N.mm$

$= 771.2 \ kN.m$

b- Compression Steel below Yield Stress

$\rho < \rho'_{cy} \longrightarrow$ Compression steel will not reach f_y

$$\rho'_b = 0.85\beta_1 \frac{f'_c}{f_y} \frac{600}{600+f_y} + \rho' \frac{f'_s}{f_y}$$

$$\rho'_b = \rho_b + \rho' \frac{f'_s}{f_y}$$

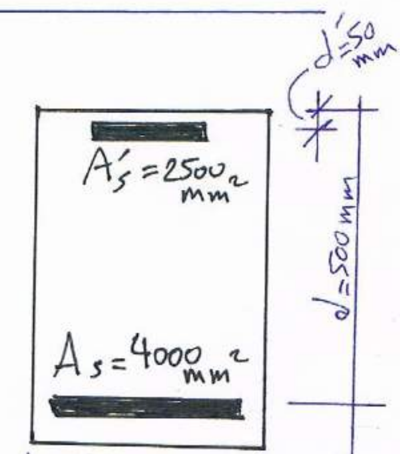
$\rho \leq \rho'_b \Longrightarrow$ Tension steel will yield

- Find (C) $\quad K_1 = \frac{A_s f_y - 600 A'_s}{0.85\beta_1 f'_c b}$, $K_2 = \frac{600 A'_s d'}{0.85\beta_1 f'_c b}$

$$C = \frac{K_1 + \sqrt{K_1^2 + 4K_2}}{2} , \quad f'_s = \frac{c-d}{c}$$

- Find $M_n = 0.85 f'_c a b \left(d - \frac{a}{2}\right) + A'_s f'_s (d - d')$

E.X. :- Find the nominal moment for cross-section of doubly rect. reinforced concrete beam shown in the figure, if, $f'_c = 30 MPa$, $f_y = 350 MPa$



Solution:-

$\rho = \frac{4000}{250 \times 500} = 0.032$, $\rho' = \frac{A'_s}{250 \times 500} = 0.02$

$\rho_b = 0.85 \beta \frac{f'_c}{f_y} \frac{600}{600+f_y} = 0.85^2 \cdot \frac{30}{350} \cdot \frac{600}{600+350}$

$\rho_b = 0.039$

$\rho'_{cy} = 0.85 \beta_1 \frac{600}{600-f_y} \frac{f'_c}{f_y} \frac{d'}{d} + \rho' = 0.0349$

$\rho = 0.032 < \rho'_{cy} = 0.0349$

∴ Compression steel will not reach fy

Check Tension Steel :-

$(\rho'_b) = ?$

find $f'_s \Rightarrow f'_s = 600 - (600 + f_y)\frac{d'}{d}$

$= 600 - (600 + 350) * \frac{50}{500} = 505 MPa > f_y = 350 MPa$

∴ $f'_s = 350 MPa$

$\rho'_b = \rho_b + \rho' \cdot \frac{f'_s}{f_y} = 0.039 * \frac{1}{1} + 0.02 = 0.059$

∵ $\rho < \rho'_b$ ∴ Tension Steel reach fy (yield)

Find (c) value

$K_1 = \frac{A_s f_y - 600 A'_s}{0.85 \beta_1 f'_c b} = \frac{4000(350) - 600*(2500)}{(0.85)^2 \cdot 30 \cdot 250} = -18.45$

$K_2 = \frac{600 A'_s d'}{0.85 \beta_1 f'_c b} = \frac{600*2500*50}{(0.85)^2 \cdot 30 \cdot 250} = 13840.83$

$$C = \frac{k_1 + \sqrt{k_1^2 + 4k_2}}{2} = \frac{-18.45 + \sqrt{(-18.45)^2 + 4 \times 13840.83}}{2} = 108.8 \text{ mm}$$

$$a = \beta_1 c = 0.85 * 108.8 = 92.5 \text{ mm}$$

$$f_s' = \frac{c - d'}{c} 600 = \frac{(108.8 - 50)}{108.8} * 600 = 324.3 \text{ MPa}$$

$$M_n = 0.85 f_c' a b (d - \frac{a}{2}) + A_s' f_s' (d - d')$$

$$= 0.85 \times 30 \times 92.5 \times 250 (500 - \frac{92.5}{2}) + 2500 \times 324.3 (500 - 50)$$

$$M_n = 632.408 * 10^6 \text{ N·mm}$$

$$= 632.4 \text{ kN·m}$$

C- Tensile steel below the yield stress

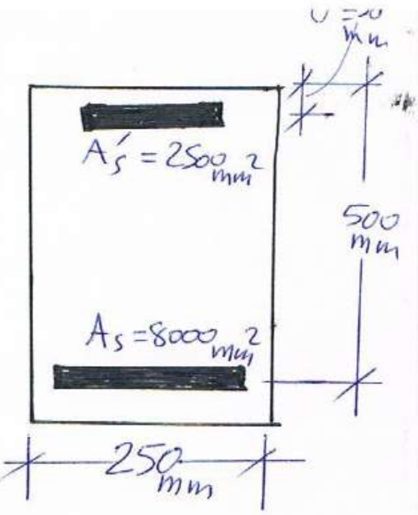
In this case $\rho > \rho_b'$ then we must find (c) by the following equation:-

$$A_s * \frac{(d-c)}{c} 600 = 0.85 \beta_1 f_c' c b + A_s' * 600 * \frac{(c - d')}{c}$$

Then find f_s', f_s

$$M_n = 0.85 f_c' a b (d - \frac{a}{2}) + A_s' f_s' (d - d')$$

E.X. :- Find the nominal moment for cross-section of doubly rectangular reinforced beam shown below, for the following data : $f'_c = 30$ MPa , $f_y = 350$ MPa



Solution :-

$$\rho = \frac{8000}{250 \times 500} = 0.064 \quad , \quad \rho' = \frac{A'_s}{bd} = 0.02$$

$$\rho_b = 0.039 \quad , \quad \rho'_{cy} = 0.0349$$

$$\rho'_b = \rho_b + \rho' = 0.059$$

$\rho = 0.064 > \rho'_b = 0.059$ ∴ The failure will be Compression failure

$\rho = 0.064 > \rho'_{cy}$ ∴ Compression steel will Reach f_y i.e. $f'_s = f_y$

Find (C) :-

$$A_s \frac{(d-C)}{C}(600) = 0.85 \beta_1 f'_c\, c\, b + A'_s f_y$$

$$8000 \cdot \frac{(500-C)}{C} \cdot (600) = (0.85)^2 * 30 * C * 250 + 2500 * 350$$

$C = 323$ mm (الحل بواسطة طريقة الدستور)

$$f_s = 600\left(\frac{500-C}{C}\right) = 600\left(\frac{500-323}{328}\right) = 328.8 \text{ MPa}$$

$$a = \beta_1 C = 0.85 * 323 = 274.6 \text{ mm}$$

$$M_n = 0.85 f'_c\, a\, b\left(d - \frac{a}{2}\right) + A'_s f_y (d - d')$$

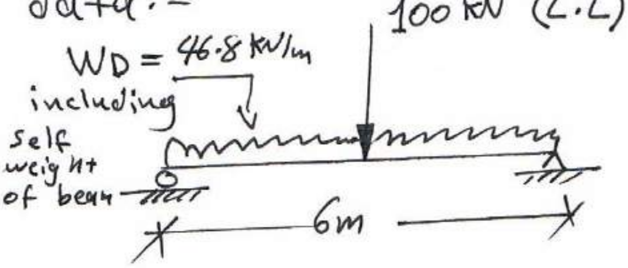
$$M_n = 1028.7 \text{ kN.m}$$

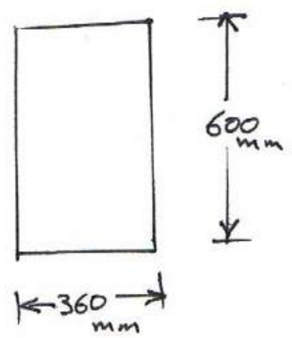
Design :-

Ex. :- For a simply supported beam shown in the Fig. shown below, Find the area of steel & its details for the following data:-

$f_y = 400$ Mpa , $f'_c = 20$ MPa

Note :- If there is need for compression steel use $d' = 65$ mm.





Solution :-

Assume 2 layers of steel Reinf.

* $d = h - 100$
  $= 500$ mm
* $P_u = 100 * 1.6 = 160$ kN
* $W_u = 46.8 * 1.2 = 56.16$ kN/m

- $M_u = \frac{P_u * L}{4} + \frac{W_u * L^2}{8}$

  $= 160 * \frac{6}{4} + 56.16 * \frac{(6)^2}{8} = 492.72$ kN.m

- from Table (P3) $\rho_{max} = 0.0155$

- $R = \frac{M_u}{\phi b d^2}$, $m = \frac{f_y}{0.85 f'_c}$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{f_y}}\right)$$

$m = \frac{400}{0.85*20} = 23.5$, $R = \frac{492.72*10^6}{0.9*360*(500)^2} = 6.08$

$\rho = \frac{1}{23.5}\left(1 - \sqrt{1 - \frac{2*23.5*6.08}{400}}\right) = 0.0198$

$\because \rho > \rho_{max} \Longrightarrow$ Design the beam as a Doubly Reinforced Beam.

- Find A_{s1}

$A_{s1} = \rho_{max}\, b\, d = 0.155*360*500 = 2790\, mm^2$

$a = \frac{A_{s1} f_y}{0.85 f'_c b} = \frac{2790*400}{0.85*20*360} = 182\, mm$

$M_{u1} = \phi M_{n1} = \phi A_{s1} f_y \left(d - \frac{a}{2}\right)$

$\because \rho > \rho_{max} \longrightarrow \therefore \rho > \rho_t$

$\therefore \phi < 0.9$

$\phi = 0.483 + 83.3\varepsilon_t = 0.483 + 83.3*0.004$
$= 0.816$

$\therefore M_{u1} = 0.816*2790*400\left[500 - \frac{182}{2}\right]*10^{-6} = 372.5\, kN.m$

- Find M_{u2}

$M_{u2} = M_u - M_{u1} = 492.72 - 372.5 = 120\, kN.m$

- Calculate the compressive Steel Reinf. Stress.

$C = a/\beta_1 = \frac{182}{0.85} \simeq 214\, mm$

$$f'_s = \frac{c - d'}{c} 600 = \left(\frac{214 - 65}{65}\right) * 600 = 418 \text{ MPa} > f_y = 400 \text{ MPa}$$

- Finding area of compressive steel reinf.

$$A'_s = \frac{M_{u2}}{\phi f'_s (d - d')} = \frac{120.22 * 10^6}{0.816 * 400 (500 - 65)} = 847 \text{ mm}^2$$

- Area of tension steel insted of comp. steel
  (مساحة حديد الشد الذي يقابل حديد الانضغاط)

$$A_{s2} = A'_s = 847 \text{ mm}^2$$

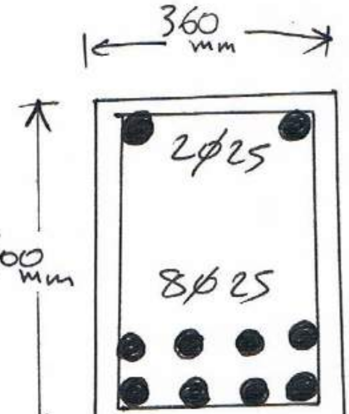
- Total Tension Steel Reinforcement

$$A_s = A_{s1} + A_{s2} = 2790 + 847 = 3637 \text{ mm}^2$$

- Use $\phi 25$ ⟶ No of bars $= \frac{3637}{\frac{\pi}{4} * (25)^2} = 7.4$

∴ Use $8 \phi 25$

$$S = \frac{360 - 100 - 4 * 25}{3} = 53 \text{ mm} > 25 \text{ mm} \quad \therefore \text{O.K.}$$



for Steel Reinf. in Comp zone

$A'_s = 847$ No of bars $= \frac{847}{491} = 1.725$

∴ Use $2 \phi 25$

.: A rectangular beam has width 250mm & Effective depth 460mm. $f_y = 300$ MPa, $f'_c = 20$ MPa. What is the maximum moment that can be utilized in design, according to the ACI Code, when: - a- $A_s = 2000\ mm^2$ b- $A_s = 5160\ mm^2$

tion :-

$$\rho_b = 0.85\beta_1\frac{f'_c}{f_y}\frac{600}{600+f_y} = (0.85)^2 * \frac{20}{300} * \frac{600}{600+300}$$

$\rho_b = 0.032$ or from Table (A3) Page 350

$$\rho = \frac{A_s}{bd} = \frac{2000}{250*460} = 0.0174 < \rho_b = 0.032$$

∴ The section is underreinforced

To calculate ϕ value we must find ρ_t

$$\rho_t = 0.85\beta_1\frac{f'_c}{f_y}\frac{0.003}{0.003+\varepsilon_t} = (0.85)^2 * \frac{20}{300} * \frac{0.003}{0.003+0.005}$$

$\rho_t = 0.018$ or from Table (A3), Page 350

$\rho = 0.0174 < 0.0180$ ∴ $\phi = 0.9$

$M_u = \phi M_n$

$$M_u = \phi\rho b d^2 f_y\left[1 - 0.59\frac{\rho f_y}{f'_c}\right]$$

$$M_u = 0.9 * 0.0174 * 250 * 460^2 * 300 * \left[1 - \frac{0.59*0.0174*300}{20}\right]$$

$= 210,253,958\ N.mm$

or

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{2000 * 300}{0.85 * 20 * 250} = 141.176 \text{ mm}$$

$$M_u = \phi A_s f_y \left(d - \frac{a}{2}\right) = 0.9 * 2000 * 300 \left(460 - \frac{141.176}{2}\right)$$
$$= 210,282,480 \text{ N.mm}$$
$$\simeq 210.282 \text{ kN.m}$$

or $M_u = 0.85 * \phi f'_c a b \left(d - \frac{a}{2}\right)$

$$= 0.85 * 0.9 * 20 * 141.176 * 250 \left(460 - \frac{141.176}{2}\right)$$
$$= 210,281,779 \text{ N.mm} \simeq 210.281 \text{ kN.m}$$

b- $A_s = 5160 \text{ mm}^2$ $\quad \rho = \frac{A_s}{bd} = \frac{5160}{250 * 460} = 0.04487 \simeq 0.045$

$\rho_b = 0.032$ $\quad \rho = 0.045 > \rho_b = 0.032$

∴ The beam is over reinforced

1- find $m = \frac{600}{0.85 \beta_1 f'_c} = \frac{600}{0.85 * 0.85 * 20} = 41.522$

$$\rho * m = 0.045 * 41.522 = 1.869$$

$$k_u = \sqrt{\left(\frac{\rho m}{2}\right)^2 + \rho m} - \frac{\rho m}{2}$$

$$= \sqrt{\left(\frac{1.869}{2}\right)^2 + 1.869} - \frac{1.869}{2} = 0.721$$

3- Find c

$$C = k_u d = 0.721 * 460 = 331.66 \text{ mm}$$
$$\simeq 331.7 \text{ mm}$$

4- Find a

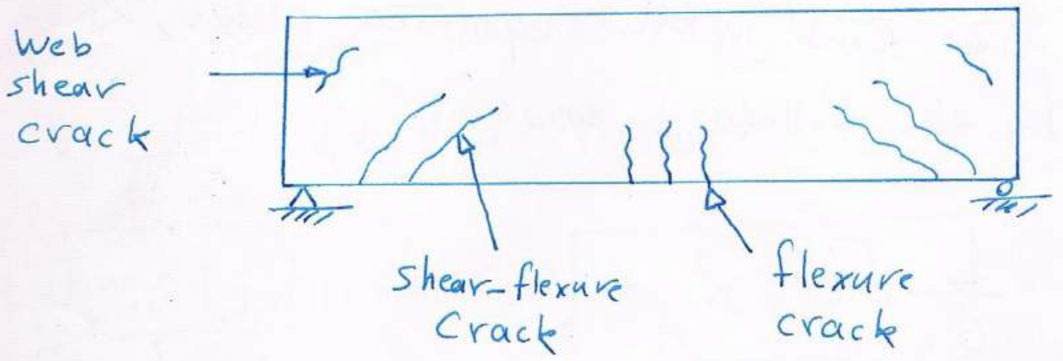
$$a = 0.85 C = 0.85 * 331.7 = 281.945 \text{ mm}$$
($0.85 \rightarrow \beta_1$)
$$a \simeq 282 \text{ mm}$$

5- Find M_n

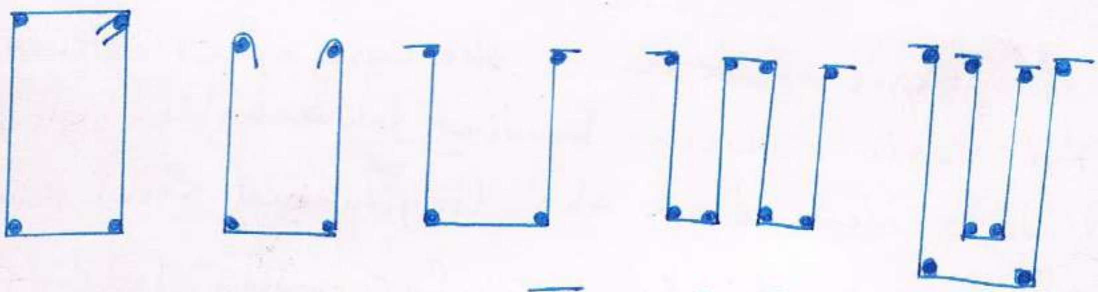
$$M_n = 0.85 f'_c * a * b \left(d - \frac{a}{2}\right)$$
$$= 0.85 * 20 * 282 * 250 * \left(460 - \frac{282}{2}\right)$$
$$= 382{,}321{,}500 \text{ N.mm}$$
$$= 382.321 * 10^6 \text{ N.mm} = 382.321 \text{ kN.m}$$

Shear & Diagonal Tension :-

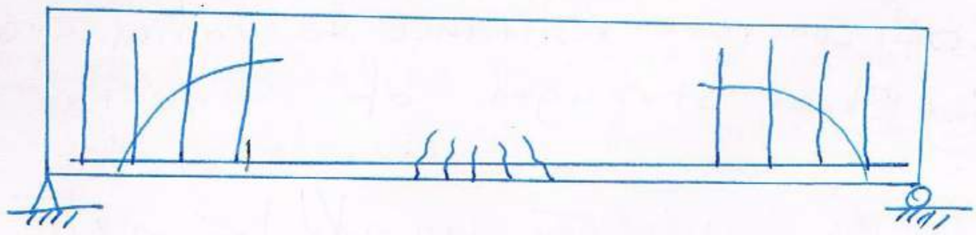
Diagonal tension stresses represent the combined effect of shear stress & bending stress.



Web Reinforcement



Different Types of Stirrups

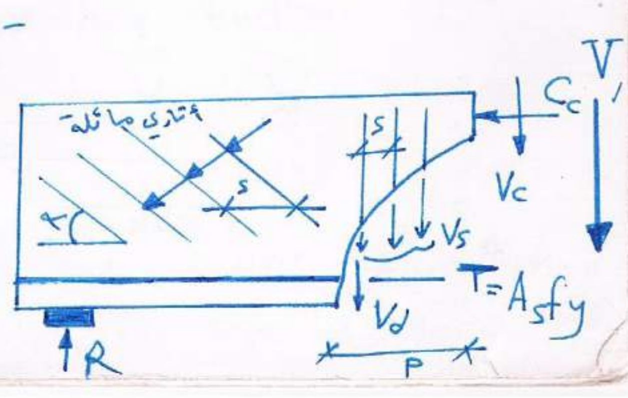


Shear Strength of Beams :-

$$V = V_d + V_s + V_c$$

(قابلية) تعشق

$$V = V_s + V_c$$



Shear Strength of Concrete :-

$$\nu_{cr} = \frac{V_{cr}}{b_w d} = 0.3\sqrt{f_c'}$$

because of reduction in area which was caused by flexural cracking, the shear strength of beam is less than that found in the equation above, & it is find by the following equation.

$$\nu_{cr} = \frac{V_{cr}}{b_w d} = \frac{1}{6}\sqrt{f_c'}$$

That means bending moment may caused a decreasing in shear strength to about half its magnitude.

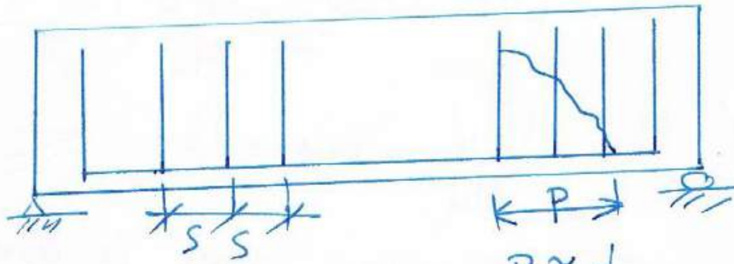
The stresses of shear in the case of cracking depend on the ratio between bending moment to shear & it is also depend on the longitudenal steel reinforcement ratio, because this steel reinforcement lead to decrease the cracks caused by bending & then increase of concrete resistance to radial cracks, i.e. increasing shear strength of concrete.

$$\nu_{cr} = \frac{V_{cr}}{b_w d} = \frac{1}{7}\left(\sqrt{f_c'} + 120\rho\frac{Vd}{M}\right) \leq 0.3\sqrt{f_c'}$$

Shear Strength of Web Reinforcement:-

$$V_s = n A_v f_y$$

№ of stirrups (n); area of shear reinforcement (A_v)

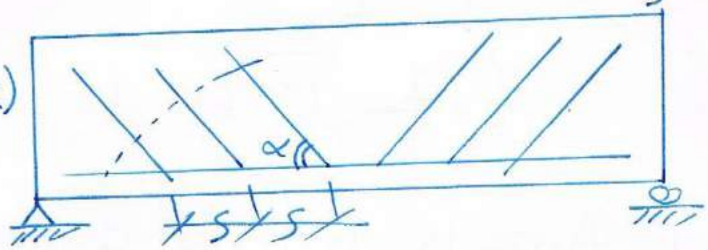


$P \simeq d$, $n = \frac{P}{S}$ or $n = \frac{d}{S}$

$$V_s = \frac{A_v f_y d}{S} \quad \text{(vertical stirrups)}$$

$$V_s = \frac{A_v f_y d \,(\sin\alpha + \cos\alpha)}{S}$$

(inclined stirrups)



$$V_n = V_c + V_s = V_c + \frac{A_v f_y d}{S}$$

$$V_u \leq \phi V_n.$$

In the case of there is no concentrated force between the face of the support & in the distance equal to (d), so the critical section for maximum shear force is taken in distance about (d) from the face of the support. The distance (S) from the face of the support to (d) is equal to the space calculated at the distance (d) from the face of the support.

If the conditions above are not occure then the critical section is taken at the face of the support.

According to ACI-code

$$V_c = \left(\sqrt{f'_c} + 120 \rho_w \frac{V_u d}{M_u}\right)\frac{b_w d}{7} \leq 0.3\sqrt{f'_c}\, b_w d$$

& the $\frac{V_u d}{M_u}$ must be ≤ 1.0

The equation above is used for researchs & Programming but for design the code give this eq. :-

$$V_c = \frac{1}{6}\sqrt{f'_c}\, b_w d$$

If there is an axial compressive force, the shear resistance will increase and can be found by this eq.

$$V_c = \left(1 + \frac{N_u}{14 A_g}\right)\left(\frac{\sqrt{f'_c}}{6}\right) b_w d$$

where:- N_u is a compressive force (N)
A_g is a total section area.

If there is an axial tension force, then, the shear resistance will decrease & can be found by the following eq. :-

$$V_c = \left(1 + \frac{0.3 N_u}{A_g}\right)\left(\frac{\sqrt{f'_c}}{6}\right) b_w d$$

where:- N_u is tension force in (N) with Negative sign (−).

Shear Design of Beams:-

a- Minimum Shear Reinforcement:-

Theoretically there is no need to shear reinforcement, when the shear force is less than concrete strength* design:-

$$V_u \leq \phi V_c$$

& the following equation is used to find shear strength of concrete

$$V_c = \frac{1}{6} \sqrt{f'_c}\, b_w\, d$$

But the ACI code requires provision of at least a minimum area of web reinforcement equal to:-

$$A_v = \frac{1}{16} \sqrt{f'_c}\, \frac{b_w S}{f_y} \geqslant \frac{b_w S}{3 f_y}$$

when $V_u > \frac{\phi V_c}{2}$

$$S_{max} \leq \begin{cases} \dfrac{16 A_v f_y}{\sqrt{f'_c}\, b_w} \\ \dfrac{3 A_v f_y}{b_w} \end{cases}$$

There is no need for shear reinforcement when

$$V_u \leq \phi \frac{V_c}{2}$$

Shear Design of Beams:-

A- Minimum Shear Reinforcement:-

Theoretically there is no need to shear reinforcement, when the shear force is less than concrete strength design:- $V_u \leq \phi V_c$

& the following equation is used to find shear strength of concrete

$$V_c = \frac{1}{6}\sqrt{f_c'}\, b_w\, d$$

But the ACI code requires provision of at least a minimum area of web reinforcement equal to:-

$$A_v = \frac{1}{16}\sqrt{f_c'}\,\frac{b_w S}{f_y} \geq \frac{b_w S}{3 f_y}$$

when $V_u > \frac{\phi V_c}{2}$

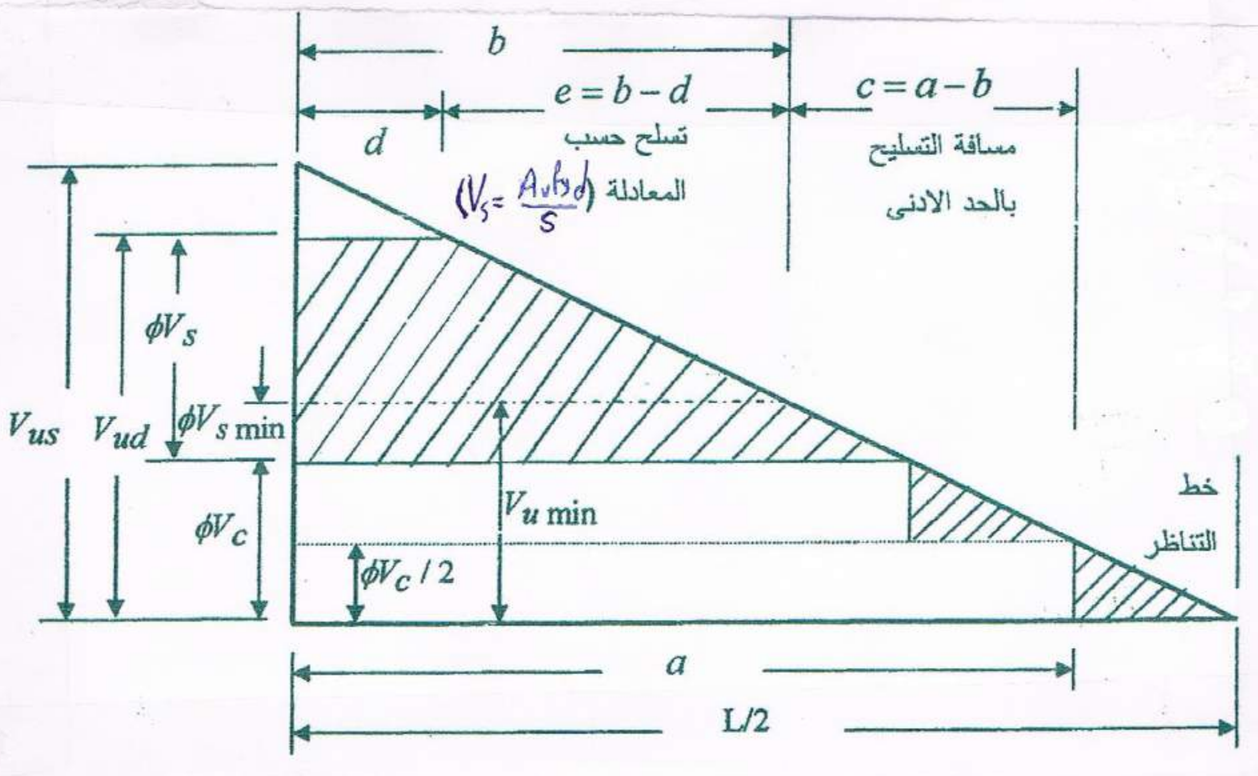
$$S_{max} \leq \begin{cases} \dfrac{16 A_v f_y}{\sqrt{f_c'}\, b_w} \\ \\ \dfrac{3 A_v f_y}{b_w} \end{cases}$$

There is no need for shear reinforcement when

$$V_u \leq \phi \frac{V_c}{2}$$

B- Region of Web Reinforcement

- When (V_{ud}) Shear force design at critical section less than ($\phi V_c/2$), there is no need to stirrups
- When (V_{ud}) larger than ($\phi V_c/2$) & less or equal (ϕV_c) then the beam must reinforced by minmum shear reinforcement for the distance varied from the face of the support to the point, that at this point the shear force equal to ($\phi V_c/2$).
- If the (V_{ud}) (shear force design) at critical section is greater than (ϕV_c), then there will be categories according to the Fig. below.



This Fig. represents the shear force diagram for a half Uniformly distributed load simply supported beam. These catogaries are:-

1- The distance between critical section and face of the support. The shear reinforcement at this distance equal to the same amount of reinforcement for critical section. That means the distance between the Stirrups at critical section (S_o). The first stirrup will be putted at distance equal to ($S_o/2$) from the support face.

2- The distance from the point reinforced with minimum shear reinforcement (b) to the critical section which is called (e) and it reinforced according to equation ($V_s = \frac{A_v f_y d}{S}$). The minmum shear reinforcement means that, the distance between Stirrups, is the maximum distance (S_{max}). The distance (b) is determined by calculating the minimum shear strength of reinforcement (i.e. $S = S_{max}$)

$$V_{s\,min} = \frac{A_v f_y d}{S_{max}}$$

After that, minmum shear strength design ($V_{u\,min}$) will calculated.

$$V_{u\,min} = \phi V_{s\,min} + \phi V_c$$

From equilibrium or the trianguls theory (b) can be found. At this point shear strength design equal to ($V_{u\,min}$).

3- The distance from the point which there is no need to shear reinf. (at distance (a) from support face) to the point, which the shear reinforcement at this point is equal to minimum reinforcement (point (c)). This distance will reinforce by minimum reinforcement ($S = S_{max}$). (a) can be found by force equilibrium or the trinanguls theory. Shear Force design at distance (a) equal to ($\phi V_c/2$)

- There is no need for shear reinforcement between point (a) and the point which, at this point the shear force equal to Zero.

C- Design of Web Reinforcement :-

* $S = (A_v f_y d)/V_s$
* $V_s = V_n - V_c = \frac{V_u}{\phi} - V_c$

$$\boxed{V_s = \frac{V_u}{\phi} - V_c}$$

* $S = \frac{A_v f_y (\sin\alpha + \cos\alpha)}{V_s}$

* When $V_s \leq \frac{1}{3}\sqrt{f'_c}\, b_w d \; (=2V_c)$

$$S_{max} \leq \begin{cases} d/2 \\ 600\,mm \\ \frac{3 A_v f_y}{b_w} \\ \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w} \end{cases}$$

* when $V_s > \frac{1}{3}\sqrt{f'_c}\, b_w d \; (=2V_c)$

$$S_{max} \leq \begin{cases} d/4 \\ 300\,mm \\ \frac{3 A_v f_y}{b_w} \\ \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w} \end{cases}$$

* If $V_s > \frac{2}{3}\sqrt{f'_c}\, b_w d \; (=4V_c)$ The Section must be changed.

Design Procedure for Web Reinforcement

1- Analayzing the beam and drow S.F. Diagram

2- Find shear force design (V_{ud}) and find (ϕV_c) from the equations below according to the kind of loadings:-

$$V_c = \frac{1}{6}\sqrt{f'_c}\, b_w d$$

$$V_c = \left(1 + \frac{N_u}{14A_g}\right)\left[\frac{\sqrt{f'_c}}{6} b_w d\right]$$

$$V_c = \left(1 + \frac{0.3 N_u}{A_g}\right)\left[\frac{\sqrt{f'_c}}{6} b_w d\right]$$

3- If • $V_{ud} \leq \phi V_c/2 \Rightarrow$ No need for shear reinforcement

• $\phi V_c/2 \leq V_{ud} \leq \phi V_c \Rightarrow$ Minmum shear reinf.

The maximum distance between stirrups is calculated by

$$S_{max} \leq \begin{cases} d/2 \\ 600\,mm \\ \dfrac{3A_v f_y}{b_w} \\ \dfrac{16 A_v f_y}{\sqrt{f'_c}\, b_w} \end{cases} \quad \text{[the min. value]}$$

The stirrups will be continued to $V_u = \phi V_c$ and after that there is no need for shear reinf.

4- If $(V_{ud} > \phi V_c)$, then find shear force design for steel (ϕV_s). If this force is greater than $(4\phi V_c)$ then the beam section must be changed, if not the distance between stirrups (Max distance) will be find from the equations below according to (V_s) value

$$S_{max} \leq \begin{cases} d/2 \\ 600\ mm \\ \dfrac{3A_v f_y}{b_w} \\ \dfrac{16 A_v f_y}{\sqrt{f_c'}\, b_w} \end{cases} \text{ the min Value} \quad \text{if } V_s \leq \frac{1}{3}\sqrt{f_c'}\, b_w d \, (=2V_c)$$

$$S_{max} \leq \begin{cases} d/4 \\ 300\ mm \\ \dfrac{3A_v f_y}{b_w} \\ \dfrac{16 A_v f_y}{\sqrt{f_c'}\, b_w} \end{cases} \text{ the min Value} \quad \text{if } V_s > \frac{1}{3}\sqrt{f_c'}\, b_w d \, (4V_c)$$

5- Calculating the distance between the stirrups at critical section (S_o), if thès distance greater or equal to (S_{max}), then the distance from support face to the point, which at this point $(V_u = \phi V_c/2)$ will found and using $(S = S_{max})$. If

$(S_o < S_{max})$, then we find the distance which, after this distance we will reinforce by minmum reinforcement, and after that we find the distance which there is no need to shear reinforcement.

6- Find the distance between Stirrups for the region between critical section and the point of min. reinf. by using the following eq.

$$S = \frac{A_v f_y d}{V_s}$$

and the distance between Stirrups will be changeed according to the methods used before.

- If the distance between Stirrups is small, then we use bigger (Ø bar) or use Stirrups with () shape.

7. Clarify the position, Kind & radius of Stirrups (Ø bar of stirrups) on the beam diagram

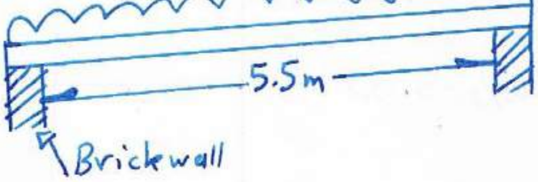
V_u		
	تغيير المقطع	
$5\phi V_c$		$V_s = 4V_c$
	S = القيمة الأقل من $\left(d/4, 300mm, \frac{3A_v f_y}{b_w}, \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w}, \frac{A_v f_y d}{V_s}\right)$	
$3\phi V_c$		$V_s = 2V_c$
	S = القيمة الأقل من $\left(d/2, 600mm, \frac{3A_v f_y}{b_w}, \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w}, \frac{A_v f_y d}{V_s}\right)$	
ϕV_c		$V_s = 0$
	S = القيمة الأقل من $\left(d/2, 600mm, \frac{3A_v f_y}{b_w}, \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w}\right)$	
$\frac{\phi V_c}{2}$		
	لا داعي لحديد التسليح	

مخطط تسليح القص حسب المقاومة
التصميمية للقص (V_u)

E.x.:- Design shear reinforcement for the beam shown below for the following data:-

b = 300 mm, d = 500 mm, LL = 40 kN/m, DL including self weight = 34 kN/m, f_y = 300 MPa, f'_c = 30 MPa.

Solution:-



* $W_u = 1.6 \times 40 + 1.2 \times 34 = 104.8$ kN/m

* finding shear force at the face of the support.

$V_{us} = 104.8 * \frac{5.5}{2} = 288.2$ kN

* Finding shear force at critical section.

$V_{ud} = V_{us} - W_u d = 288.2 - 0.5 \times 104.8 = 235.8$ kN

* $\phi V_c = 0.75 \left(\frac{1}{6} \sqrt{f'_c} \times b \times d\right)$

$= 0.75 \left(\frac{1}{6} \sqrt{30} \times 300 \times 500\right) * 10^{-3} = 102.698$ kN

Check if there is need for shear reinforcement

$V_{ud} = 235.8 > \phi V_c = 102.698$ ∴ there is a need for shear reinforcement

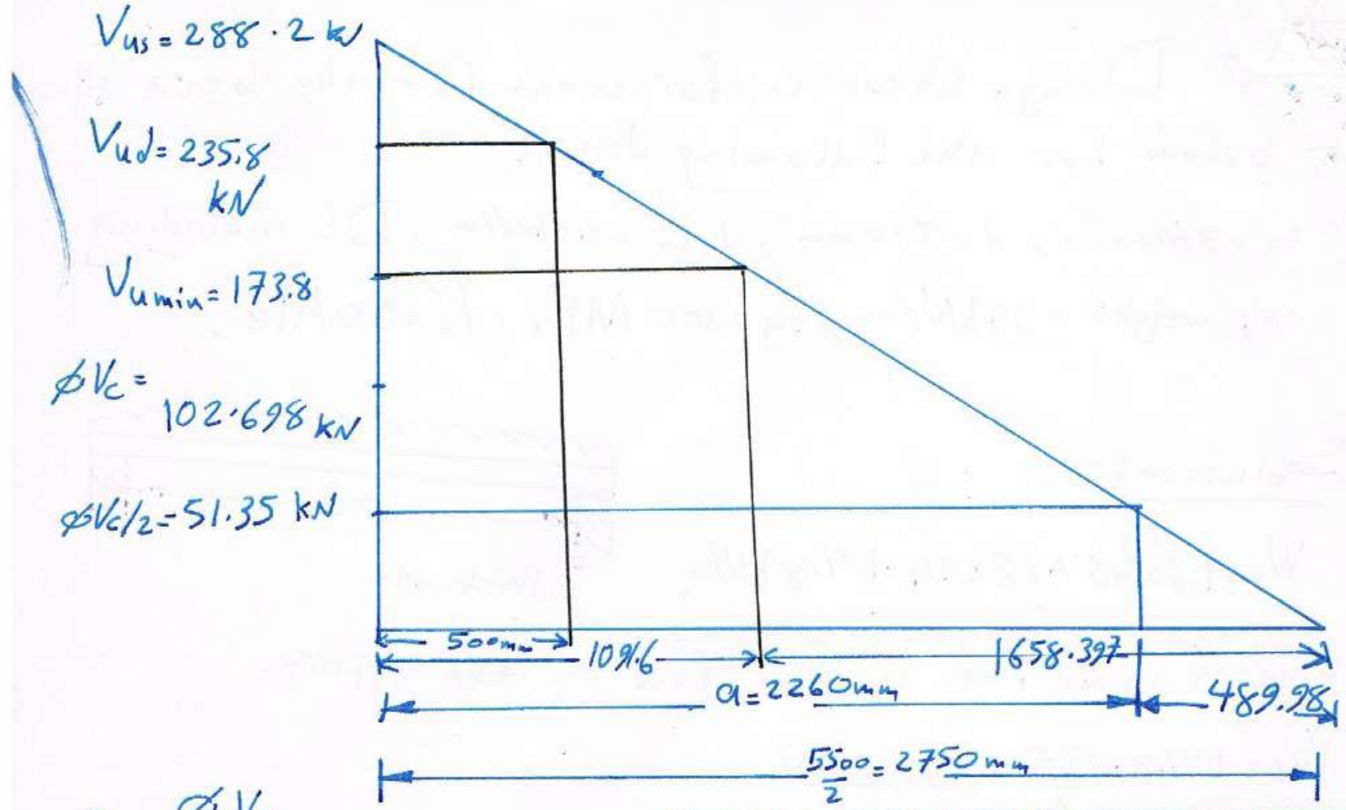
* $\phi V_s = V_{ud} - \phi V_c = 235.8 - 102.698 = 133.1$ kN

$V_s = \frac{133.1}{\phi} = \frac{133.1}{0.75} = 177.47$ kN

* $4 * \phi V_c = 4 * 102.698 = 410.792$ kN

∵ $\phi V_s < 4 \phi V_c$
$133.1 \text{ kN} < 410.792 \text{ kN}$

∴ The section is adiquate for shear resistance.



* $\frac{\phi V_c}{2} = \frac{102.698}{2} = 51.35$ kN

$\because \phi V_s < 4\phi V_c$

* $\therefore S_{max} \leq \begin{cases} d/2 = 250\,mm \checkmark \\ 600\,mm \\ \frac{3A_v f_y}{b_w} = \frac{3 \times 2 \times 79 \times 300}{300} = 474\,mm \\ \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w} = \frac{16 \times 2 \times 79 \times 300}{\sqrt{30} \times 300} = 461\,mm \end{cases}$

$\therefore$ Use $S_{max} = 250$ mm

* Find spacing of reinforcement at critical section.

$S_o = \frac{A_v f_y d}{V_s} \Longrightarrow$ use ϕ 10mm for stirrups.

$\therefore A_v = 2 * \frac{\pi}{4} \times 10^2 = 157.0\,mm^2$

$S_o = \frac{157 * 300 * 500}{177.47 * 10^{-3}} = 132.7\,mm < S_{max} = 250\,mm$

$\therefore$ Use $S_o = 130$ mm c/c

* Finding the distance in which there is no need for reinf.

$\frac{x}{51.35} = \frac{2750}{288.2} \Rightarrow x = 489.98$ mm

$a = 2750 - 489.98 = 2260$ mm

- We can find the distance (a) by other method.

$V_{us} - W_u * a = \phi V_c/2$

$288.2 - 104.8 * a = 51.35 \Rightarrow a = \frac{51.35 - 288.2}{-104.8}$

$\therefore a = 2.260$ m

* Determine the distance, which it is after reinforced by minimum reinforcement (shear reinforcement).

$\phi V_{s\,min} = \frac{\phi A_v f_y d}{S_{max}} = \frac{0.75 * 157 * 300 * 500}{250} * 10^{-3} = 70.650$ kN

$V_{u\,min} = \phi V_{s\,min} + \phi V_c = 71.1 + 102.7 = 173.8$ kN

$V_{us} - W_u b = 173.8$

$288.2 - 104.8 * b = 173.8 \Rightarrow b = \frac{173.8 - 288.2}{-104.8}$

$b = 1.0916$ m

or $\frac{x}{173.8} = \frac{2750}{288.2} \Rightarrow x = 1658.397 \Rightarrow x_1 = 2750 - 1658.397$

$\Rightarrow x_1 = 1091.6$ mm

* Distribution of shear reinforcement along the beam

a- Put the first stirrups at a distance equal to $\frac{S_o}{2} = \frac{130}{2} = 65$ mm

$\therefore$ Put the first stirrups at a distance equal to 60 mm from the face of the support.

b- Number of the stirrups (130mm)

$n = \frac{1092 - 60}{130} = 7.938$ Use 8 ϕ10 stirrup @ 130 mm c- So, the distance from the face of the support which reinforced for shear until now equal to:-

$60 + 8 * 130 = 1100$ mm

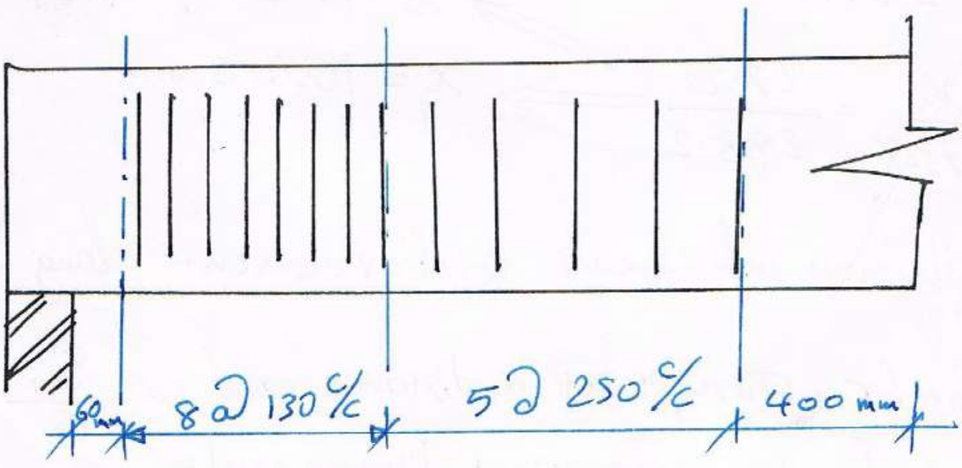
$\therefore$ No of stirrups of 250mm c/c $= \frac{2260 - 1100}{250}$

$= 4.64$

$\therefore$ use $5 \phi 10_{mm}$ stirrups @ 250 c/c

So, the space which reinforced to shear equal to $1100 + 5(250) = 2350$ mm

So, the region which not reinforced for shear is equal to $2750 - 2350 = 400$ mm



or from equilibrium

$V_{us} = W_u a = \phi V_c/2$

$288.2 - 104.8a = 51.35\ kN \Longrightarrow a = 2260\ mm$

finding the distance which after this distance the shear reinforcement in minmun magnitude

$$\phi V_{s\,min} = \frac{\phi A_v f_y d}{S_{max}} = \frac{0.75 \times 2 \times 79 \times 300 \times 500 \times 10^{-3}}{250}$$

$= 71.1\ kN$

$V_{u\,min} = \phi V_{s\,min} + \phi V_c = 71.1 + 102.7 = 173.8\ kN$

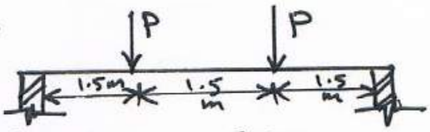
$V_{us} - W_u b = 288.2 - 104.8b = 173.8 \Rightarrow b = 1092\ mm$

the distance between critical Section & the point of minmum shear reinf. is small

$e = b - 500 = 1092 - 500 = 592\ mm$

So use the same shear reinf. for critical section

Ex.:- A reinforced concrete girder with a rectangular section, loaded by two concentrated loads, each of them consist of 80kN service Live load & 60 kN service dead load. The width of this girder equal to 300mm & its effective depth equal to 550mm. Design this girder for shear.



Solution:- 1- $h = 550 + 100 = 650$ mm (if we assume the reinforcement in 2 layrs)

$\therefore W_g = 0.65 \times 0.3 \times 24 = 4.68$ kN/m

$W_u = 4.68 \times 1.2 = 5.62$ kN/m

$P_u = 1.2 * 60 + 1.6 * 80 = 200$ kN

$V_{us} = 200 + 4.5 \times \frac{5.62}{2} = 212.65$ kN

2- Calculate V_{ud}

$V_{ud} = 212.65 - 5.62 \times 0.55 = 209.6$ kN

$\phi V_c = 0.75 (\frac{1}{6}) \sqrt{30} \times 300 \times 550 * 10^{-3} = 112.96$ kN

$\frac{\phi V_c}{2} = 56.48$ kN

3- $V_{ud} = 209.6$ kN $> \phi V_c = 112.96$ kN $\therefore$ There is need for shear reinf.

calculate $\phi V_s = 209.6 - 112.96 = 96.6$ kN

$\therefore V_s = 128.8$ kN

$\because \phi V_s < 4 \phi V_c$ $\therefore$ the section is adiqute for shear.

$\because \phi V_s < 2 \phi V_c$

Use ϕ bar 10mm for strirups

$\therefore S_{max}$
- 600mm
- $d/2 = 275$ mm
- $\frac{3 A_v f_y}{b_w} = 3 * 2 * \frac{\pi}{4} * 10^2 \times 300 = 474$ mm
- $\frac{16 A_v f_y}{\sqrt{f_c'}\, b_w} = \frac{16 * 2 \times 79 \times 300}{\sqrt{30} \times 300} = 462$ mm

Distance from (0 – 1.5) m

- from shear force diagram

$V_u = 204.22 > \phi V_c = 112.96$ kN

∴ all the distance will reinforced for shear

$$\phi V_{s\,min} = \frac{\phi A_v f_y d}{S_{max}} = \frac{0.75 \times 2 \times 79 \times 300 \times 550}{275} \times 10^{-3} = 71.1 \text{ kN}$$

$V_{u\,min} = 71.1 + 112.96 = 184.06$ kN

$V_{u\,min} < 204.22$ kN

So, we don't use $S = S_{max}$

Distance (3 – 1.5) m

$V_u = 4.22 < \phi V_c / 2 = 56.48$ kN

∴ There is no need for shear Reinf.

Note :- (Because the variation in shear in the region (0 – 1.5) is small, the distance between the stirrups is still with the same reinforcement for shear for S_o)

Put the firs stirrup in the distance equal to $S_o/2 = \frac{200}{2} = 100$ mm from the face of the support

So, the No other stirrups

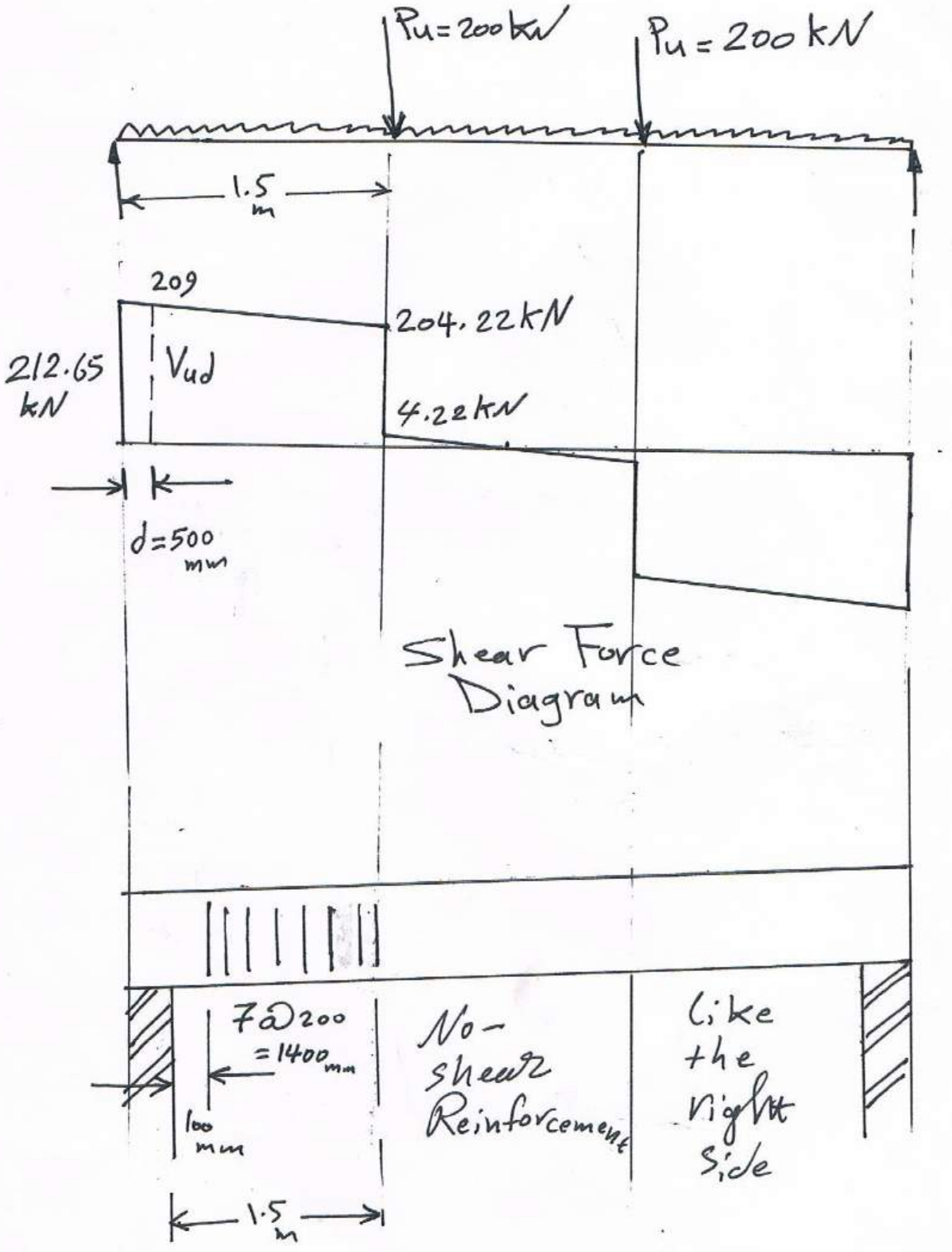
$$n = \frac{1500 - 100}{200} = 7$$

i.e. (7 @ 200 mm c/c)

Use $S_{max} = 275mm$

5- Calculate (S_o) $$S_o = \frac{A_v f_{yd}}{V_s} = \frac{2 * \frac{\pi}{4} * 300 * 550}{128.8 * 1000} = 202 \, mm$$

USE ⟹ $S_o = 200mm < S_{max}$



6.5

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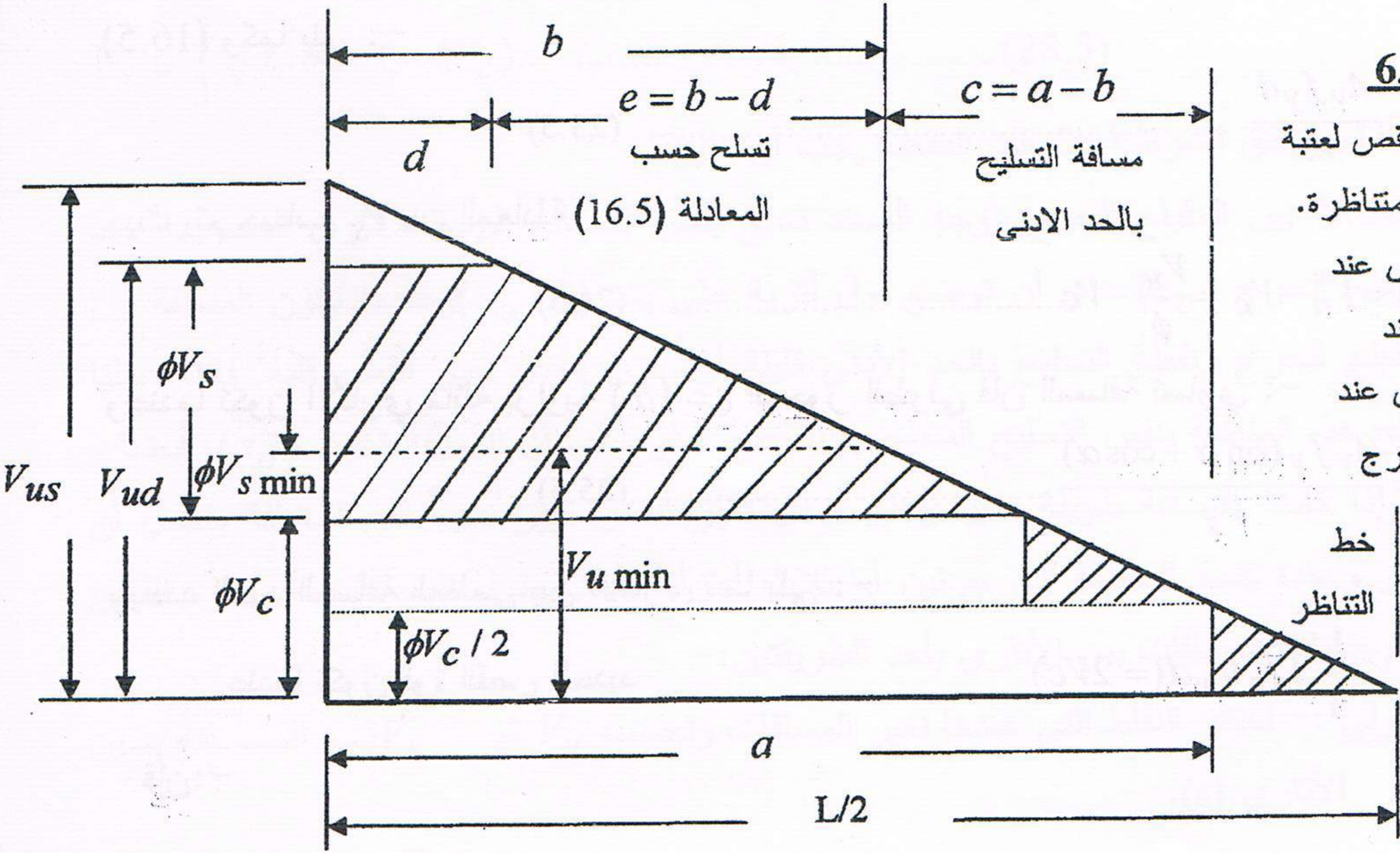
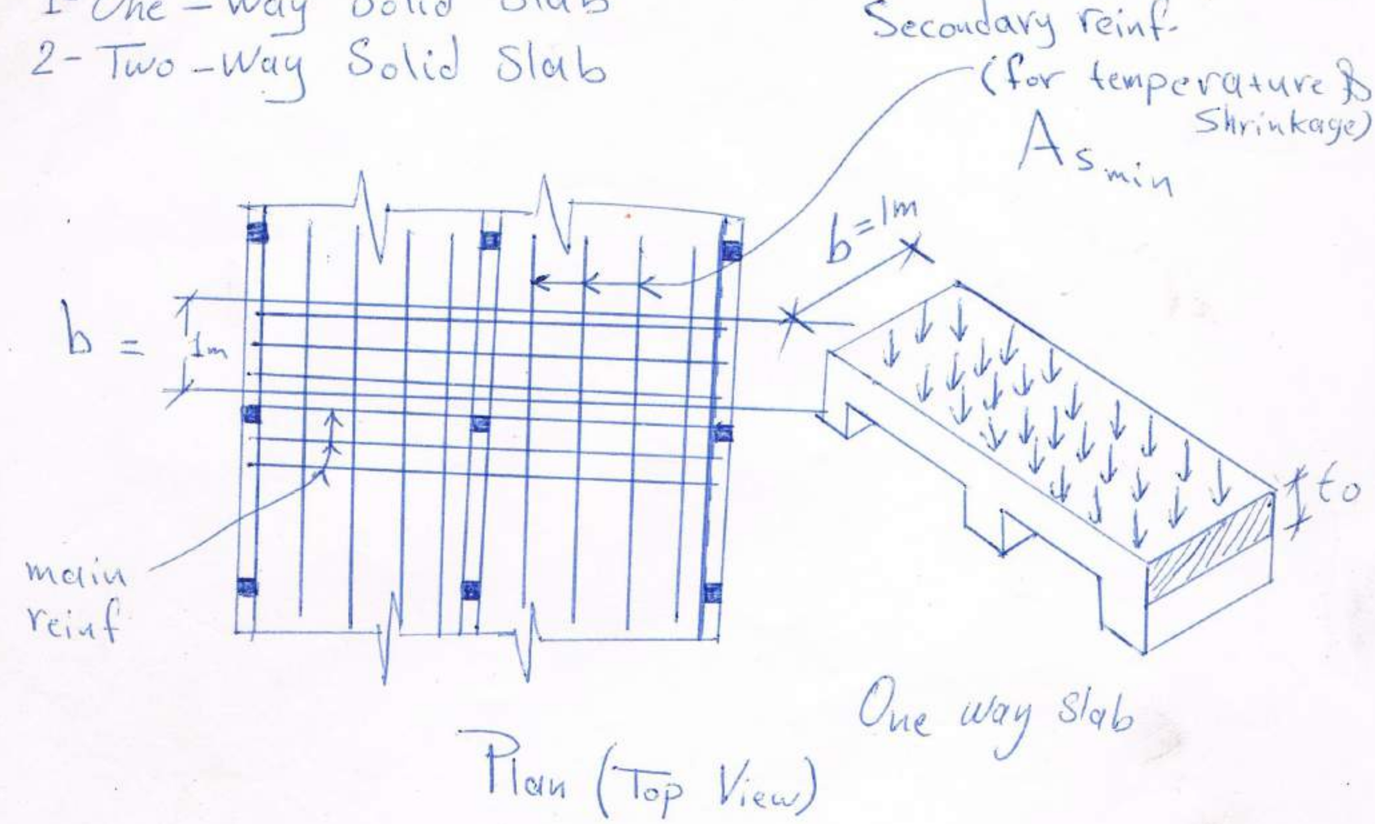
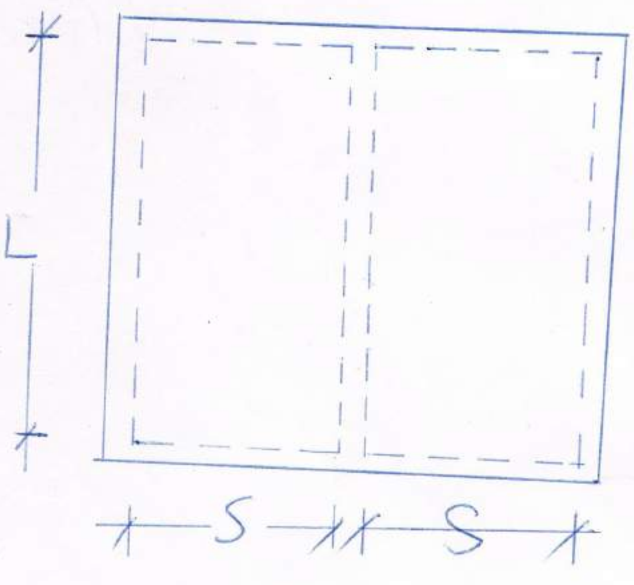


Reinforced Concrete Slabs

1- One-Way Solid Slab
2- Two-Way Solid Slab





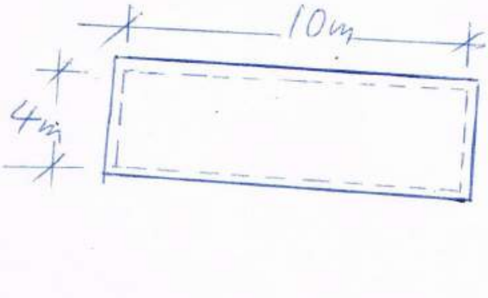
If $\frac{L}{S} > 2$ One-way slab

$\frac{L}{S} \leq 2$ Two-way slab

$m = \frac{S}{L} < 0.5$ one-way

≥ 0.5 Two-way

E.x.



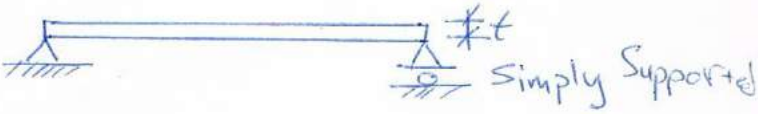
Since $\frac{10}{4} = 2.5 > 2$ $\therefore$ one way

Slab Thickness

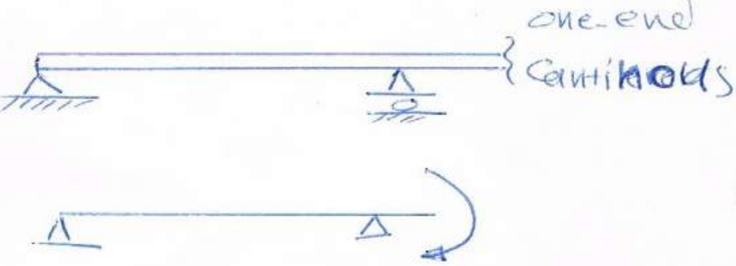
Minumum thickness of one way slab for deflection Control

$$t_{min} = \frac{L}{10}$$

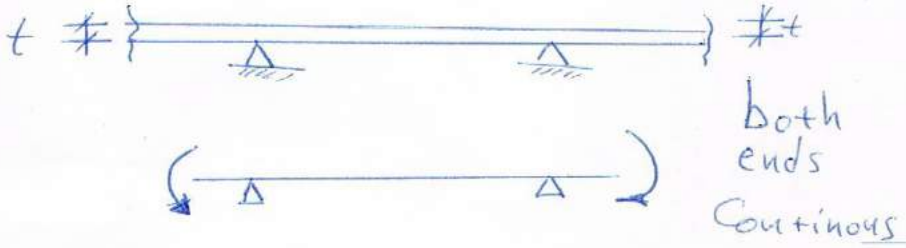
$$t_{min} = \frac{L}{20}$$



$$t_{min} = \frac{L}{24}$$

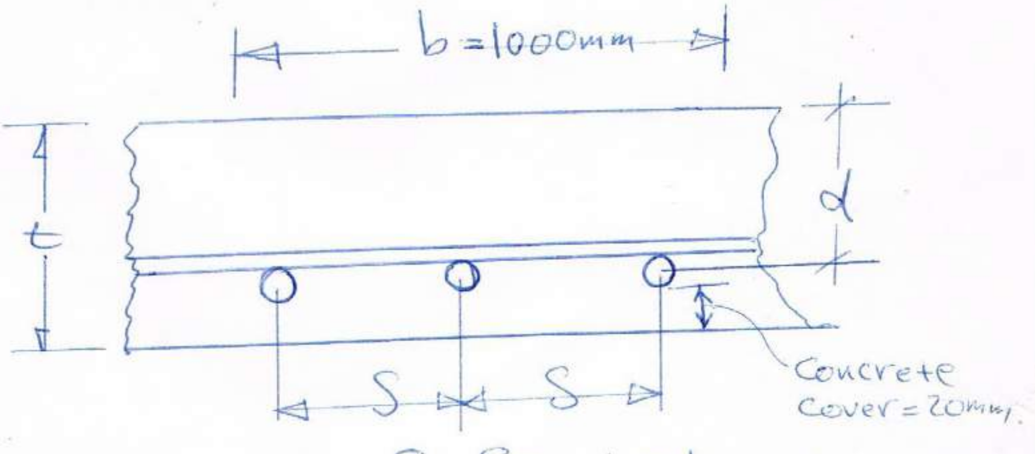


$$t_{min} = \frac{L}{28}$$



$$d = t - 20 - \frac{d_b}{2}$$

$d_b = 10_{mm}, 12_{mm}, 16_{mm}$



S = Spacing between bars

Concrete cover	
20 mm	slab
40 mm	beam, column
75 mm	foundation

Check the effective depth according to shear requirements

* The design shear strength (ϕV_c) must be equal or greater than design shear force at critical section, if not, the (h) must be greater.

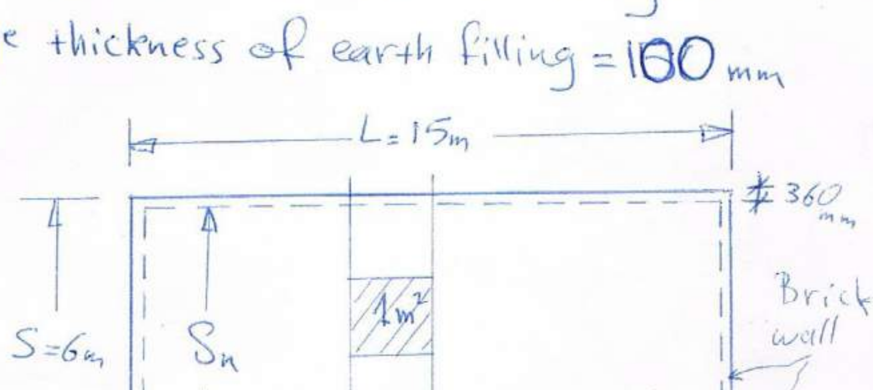
Ex. Design the slab (Roof) showing in the Fig. below:-
The slab is simplly supported, $f_y = 300\ MPa$, $f'_c = 20MPa$.

$WL = 1.5\ kN/m^2$, Tiles $= 1.0\ kN/m^2$, Earth filling $= 2\ kN/m^2$
assuming the average thickness of earth filling = 100 mm

Solution:-

$$* \frac{L}{S} = \frac{(15-2*0.36)}{(6-2*0.36)} = 2.70 > 2$$

∴ The slab is one way slab.



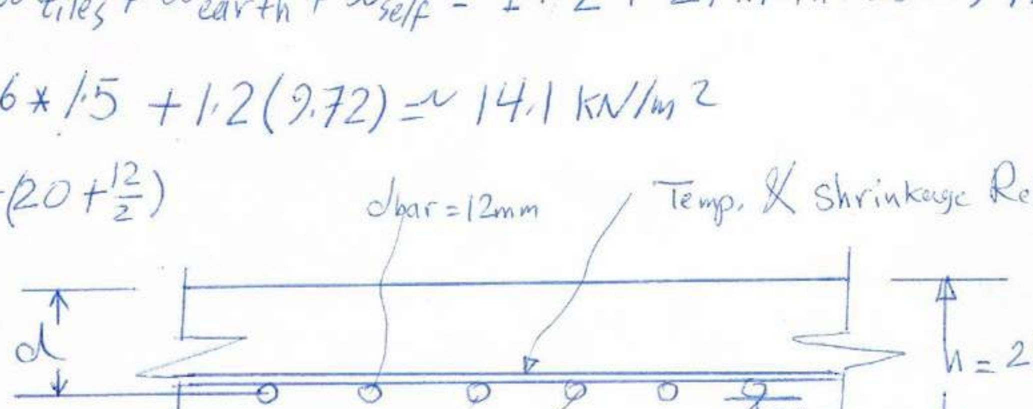
$$* \; h = \frac{6-2*0.36}{20} * 1000 = 264_{mm} \quad \text{use } h = 280mm$$

$$* \; W_D = W_{tiles} + W_{earth} + W_{self} = 1 + 2 + 24 \times 1 \times 1 \times 0.28 = 9.72\ kN/m^2$$

$$W_u = 1.6 * 1.5 + 1.2(9.72) \simeq 14.1\ kN/m^2$$

$$d = 280 - (20 + \tfrac{12}{2})$$

$$d = 254mm$$



* Checking for shear

$V_{us} = \frac{14.1 * 5.28}{2} = 37.22\ kN$

$V_{ud} = V_{us} - W_u d = 37.22 - 14.1 * 0.254 = 33.643\ kN$

$\phi V_c = 0.75 * \frac{1}{6} \sqrt{f'_c} * b * d = 0.75 * \frac{1}{6} \sqrt{20} * 1000 * 254 * 10^{-3}$

$\phi V_c = 142\ kN > V_{ud} = 33.64\ kN$

∴ The thickness is adiquate for shear.

* Bending Moment :-

$M_{u_{max}} = \frac{W_u\ S_n^2}{8} = \frac{14.1 * (5.28)^2}{8} = 49.14\ kN.m$

$\rho_{max} = 0.0206$

$M_u = \phi \rho f_y b d^2 (1 - \frac{0.59 \rho f_y}{f'_c})$

$49.14 * 10^6 = 0.9 * 300 * \rho * 1000 * 254^2 (1 - 0.59 * \frac{\rho * 300}{20})$

$49.14 * 10^6 = 1.742 * 10^{10} \rho - 1.542 * 10^{11} \rho^2$

$\rho^2 - 0.113 \rho + 0.00031 = 0$

$\rho = \frac{-0.113 \pm \sqrt{0.113^2 - 4 \times 1 \times 0.00031}}{2}$

$\rho = 0.0028 < \rho_{max} = 0.0206$

$A_{s_{min}} = 0.002\ bh = 0.002 * 1000 * 280 = 560\ mm^2/m$

$A_s = \rho b d = 0.0028 * 1000 * 254 = 711.2\ mm^2/m > A_{s_{min}}$

$\rho_t = 0.018 > \rho$ ∴ $\phi = 0.9$ ok

$A_{s\,bar} = \frac{\pi}{4} \times 12^2 = 113\ mm^2$, $S_{max} \leq \begin{cases} 450\ mm \checkmark \\ 3h = 3 \times 280 = 840\ mm \end{cases}$

$S = \frac{1000}{712/113} = 158.7\ mm\ c/c$ use $S = 150\ mm\ c/c < 450\ mm$

A_s for Temperature & shrinkage

$A_s = A_{s\,min} = 560\ mm^2/m$

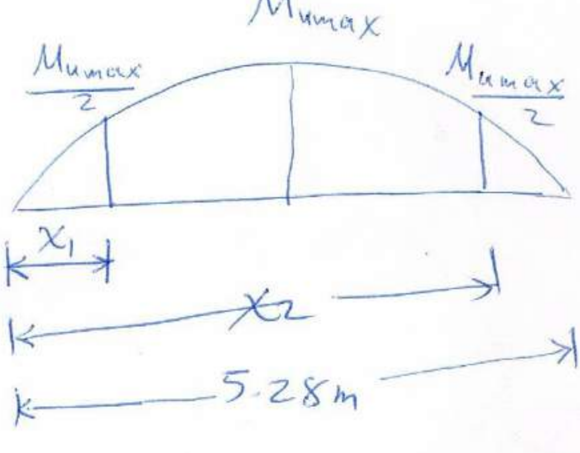
$S = \frac{1000}{560/113} = 201.8$ $S_{max} \leq \begin{cases} 5h = 1400\ mm \\ 450\ mm \checkmark \end{cases}$

$\therefore$ Use $S = 200\ mm\ c/c$

نقرر قطع نصف حديد التسليح

The B.M. value in any Point at distance $= x$

is $M_x = \frac{w s_n}{2} * x - \frac{w x^2}{2}$



$\left(\frac{49.14}{2}\right) = \frac{14.1 * 5.28}{2} * x - \frac{14.1\,x^2}{2}$

$24.57 = 37.224x - 7.05x^2$

$x^2 - 5.28x + 3.485 = 0$

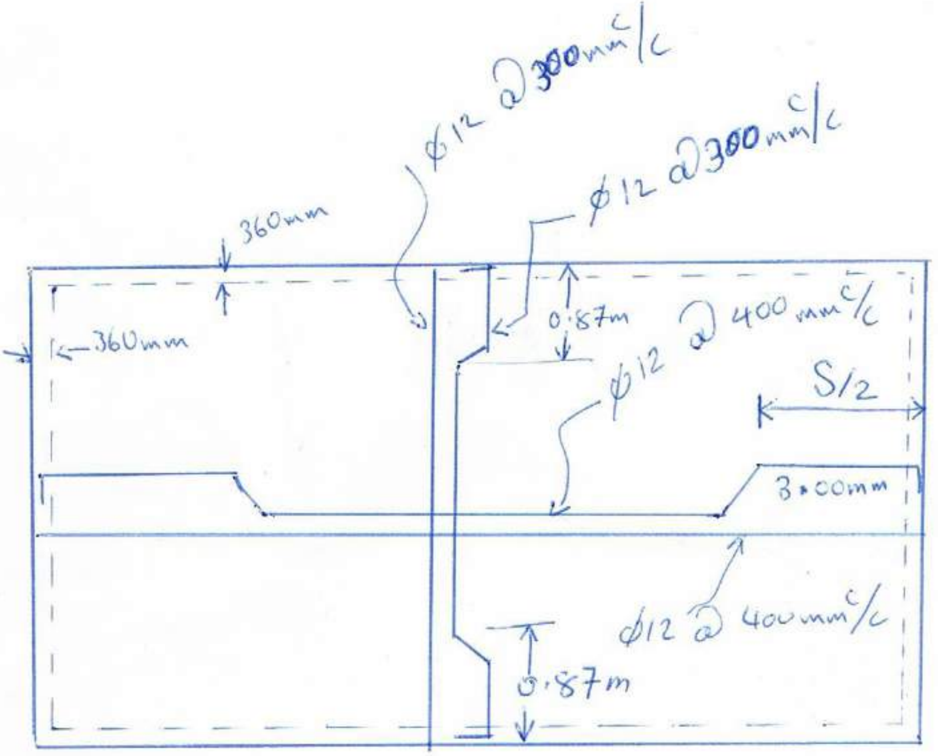
$x = \frac{5.28 \pm \sqrt{5.28^2 - 4 \times 1 \times 3.85}}{2}$

$x_1 = \frac{1.748}{2} = 0.874\ m$, $x_2 = \frac{8.812}{2} = 4.41\ m$

نقاط العوم الحقيقية .

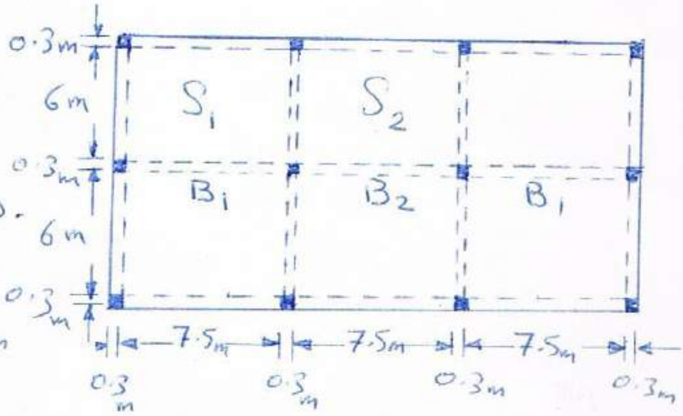
$$x_{1\,Real} = X_1 - \frac{d}{2} = 0.874 - \frac{0.254}{2} = 0.747\,m$$

$$x_{2\,Real} = X_2 + \frac{d}{2} = 4.41 + \frac{0.254}{2} = 4.537\,m$$



Dr.NAHLA.N HILA

Ex:- Design the slabs of the building showing below, $L.L = 3 kN/m^2$, The additive dead load (finshing & Tiling) equal to $2 kN/m^2$, $f'_c = 30 MPa$, $f_y = 400 MPa$. Find the magnitude of loads which applied from the slabs on the beam B_1.



Solution:-

1- Thickness of the slab.

$$h \geq \begin{cases} \frac{P}{180} = \frac{2(6000+7500)}{180} = 150\ mm \\ 90\ mm \end{cases}$$

$\therefore$ Use $h = 150\ mm$

2- Calculate the design loads:-

$D.L. = 0.15 \times 1 \times 1 \times 24 + 2 = 5.6\ kN/m^2$

$W_u = 1.2(5.6) + 1.6(3) = 11.52\ kN/m^2$

3- Cheack the thickness of slab according to shear requirements.

$$d = 150 - \underset{cover}{20} - \underset{db/2}{6} = 124\ mm$$

$$V_{ud} = 11.52\left(\frac{6}{2} - 0.124\right) = 33.13\ kN$$

$$\phi V_c = \frac{0.75}{6}\sqrt{30} * 1000 * 124 * 10^{-3} = 84.9\ kN$$

$\because V_{ud} < \phi V_c \Longrightarrow \therefore$ the thickness is adiqute for shear

4- Max. & Min. ratios of Steel Reinforcement:

$$\rho_{max} = (0.85)^2 * \frac{30}{400} * \frac{0.003}{0.003+0.004} , \varepsilon_t = 0.004 = 0.0232$$

$$A_{smin} = 0.0018bh = 0.0018 * 1000 * 150 = 270 \, mm^2/m$$

5- Calculating of BM_s.

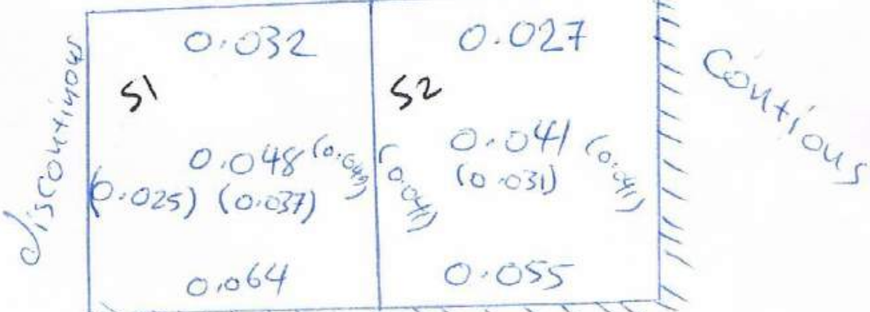
$$S \leq \begin{cases} c/c \text{ of short span length} = 6 + 0.3 = 6.3m \\ \text{clear length of short span} + 2h = 6 + 2*0.15 = 6.3m \end{cases}$$

take the smallest value $\therefore S = 6.3m$

$$L \leq \begin{cases} 7.5 + 0.3 = 7.8 \, m \\ 7.5 + 2 \times 0.15 = 7.8 \, m \end{cases}$$

$\therefore L = 7.8m$

$$m = \frac{6.3}{7.8} = 0.808$$



* Bending Moments in short direction

- Slab S1

$M_{u\,disc.}^{-} = 0.032 * 11.52 * 6.3^2 = 0.032 * 457.23 = 14.63$ kN.m/m

$M_u^{+} = 0.048 * 457.23 = 21.95$ kN.m/m

$M_{u\,cont.}^{-} = 0.064 * 457.23 = 29.26$ kN.m/m

- Slab S2

$M_{u\,disc}^{-} = 0.027 * 457.23 = 12.35$ kN.m/m

$M_u^{+} = 0.041 * 457.23 = 18.75$ kN.m/m

$M_{u\,con}^{-} = 0.055 * 457.23 = 25.15$ kN.m/m

* Bending Moments in long direction.

- Slab S1

$M_{u\,disc}^{-} = 0.025 * 457.23 = 11.43$ kN.m/m

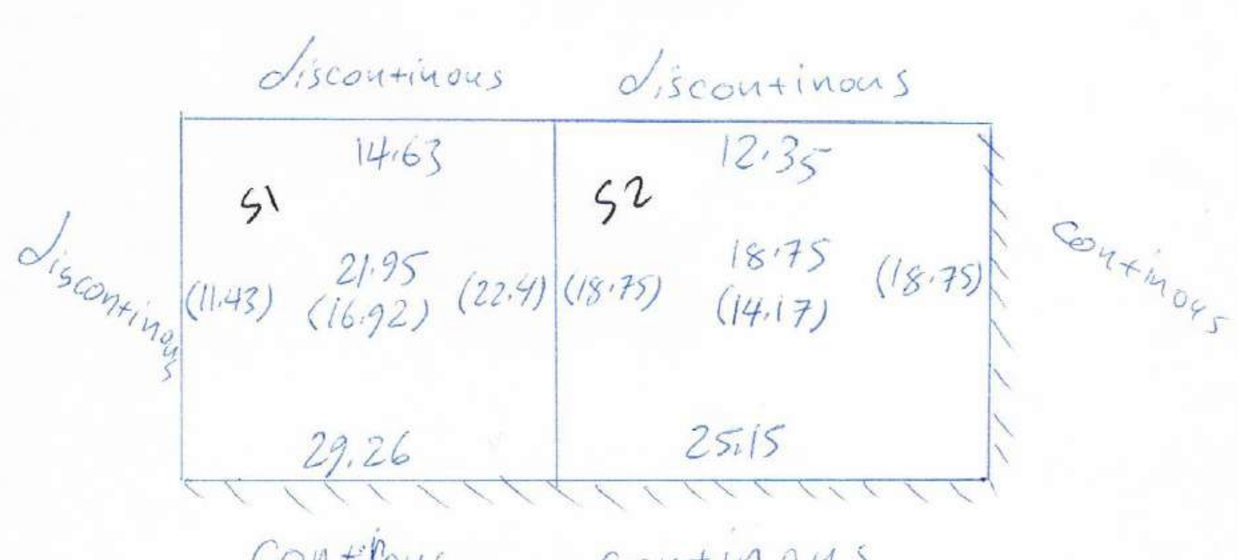
$M_u^{+} = 0.037 * 457.23 = 16.92$ kN.m/m

$M_{u\,cont.}^{-} = 0.049 * 457.23 = 22.4$ kN.m/m

- Slab S2

$M_{u\,cont.}^{-} = 0.041 * 457.23 = 18.75$ kN.m/m

$M_u^{+} = 0.031 * 457.23 = 14.17$ kN.m/m



Steel Reinforcement

a- Short Direction:-

$K = \frac{M_u}{f'_c b d^2}$, $\omega = \rho \frac{f_y}{f'_c}$, $\rho =$ [from Table (p4)]

$K = \frac{M_u}{\phi f'_c b d^2}$

Reinforcement for middle Strip in short direction (S1)

Moment	K	ω	ρ	$A_s = \rho b d$	A_s Provided	A_s added
$M_u^+ = 21.95$	0.053	0.055	0.00413	512	ϕ10/150 (526)	——
$M_u^-{}_{disc.} = 14.63$	0.035	0.036	0.0027	335	ϕ10/300 (263)	ϕ 8/300 (167)
$M_u^-{}_{con.} = 29.26$	0.070	0.074	0.0056	695	ϕ 10/150 (526)	ϕ 10/300 (263)

$\rho_{max} = 0.0232$, $\rho_t = 0.0203$, $A_{s\,min} = 0.002bh$

$A_{s\,min} = 0.00180 * 1000 * 150 = 270.0 mm^2/m$

* All ρ value $< \rho_{max} = 0.0232$ ∴ o.k.
* All ρ value $< \rho_t = 0.0203 \Longrightarrow \phi = 0.9$
* All $A_s > A_{s\,min} =$

$S = \frac{1000}{A_s / A_b} = \frac{1000 * 79}{512} = 154\,mm$

Use $S = 150\,mm$ c/c

50% of bars will be bend & then (A_s provided) is found

$A_s = \frac{1000}{S} A_b$

Reinforcement for middle strip in short direction (S_2)

Moment	K	ω	ρ	$A_s = \rho b d$ mm^2	A_s provided mm^2	A_s add. mm^2
$M_u^+ = 18.75$	0.045	0.047	0.0035	434	ϕ10/180 (438)	—
$M_{u\,disc.}^- = 12.35$	0.030	0.031	0.0023	286	ϕ10/360 (219)	ϕ8/360 (139)
$M_{u\,con.}^- = 25.15$	0.060	0.063	0.0047	583	ϕ10/180 (438)	ϕ10/360 (219)

All value of ρ & A_s is ok for $\rho_{max.}$, ρ_t & $A_{s\,min}$.

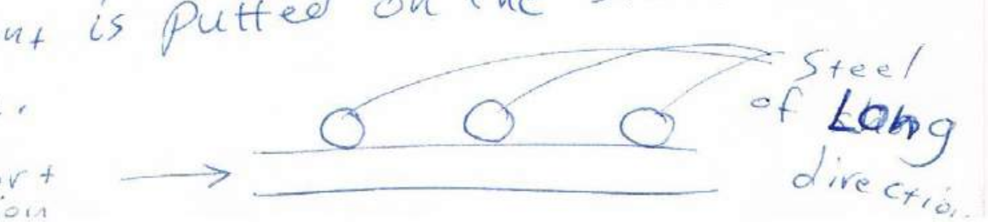
- for vertical strip the steel reinforcement = $\frac{2}{3}$ of steel reinforcement for middle strip.

* Spacing of vertical strip = 1.5 * S of middle strip.
* S must not be more than (2h).
* A_s for vertical strip $\geq A_{s\,min}$.
* In practical case $A_{s\,\text{vertical strip}} = A_{s\,\text{middle strip}}$ but in that case there are lost in steel reinforcement.

- Steel Reinforcement in Long Direction:-

$d = h - (1.5 d_b + 20) = 150 - (1.5 * 12 + 20) = 112\ mm$

This Reinforcement is putted on the steel of short direction.



Reinforcement of Long Direction.

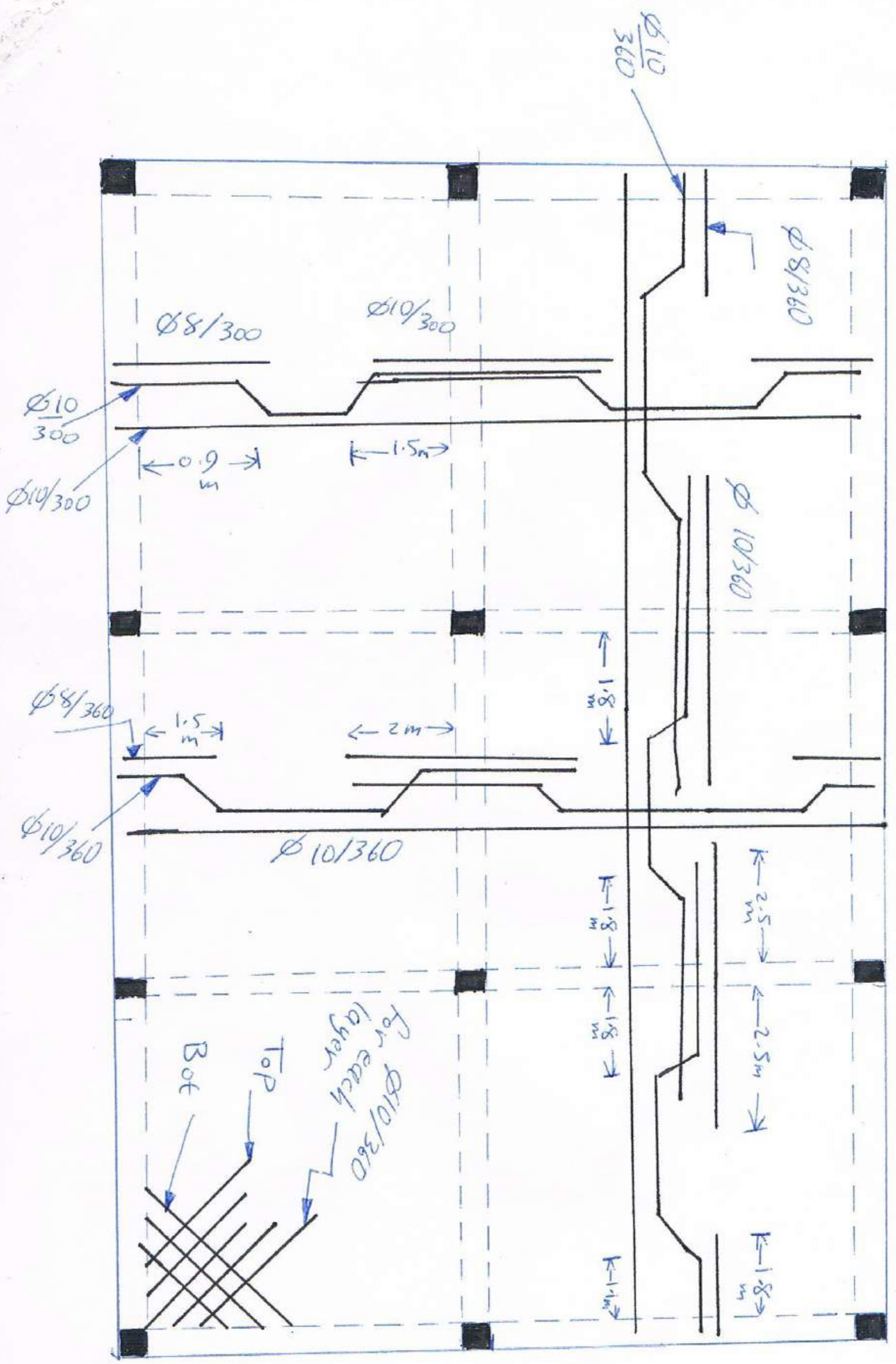
Moment KN.m	k	ω	ρ	$A_s=\rho bd$	A_s provided	A_s add.
$M_u^+=16.92$	0.05	0.052	0.0039	437	ϕ10/180 (438)	—
$M_{u\,disc.}^-=11.43$	0.034	0.035	0.0026	292	ϕ10/360 (219)	ϕ8/360 (139)
$M_{u\,cau.}^-=22.4$	0.066	0.069	0.0052	583	ϕ10/180 (438)	ϕ10/360 (219)

* For Torsion Reinforcemen the same diameters & Spacing of Posotive Reinforcement in short direction can be used.

- The Load which applied on beam (B_1) equal to:-

$$W_e=\frac{Ws}{3}\,\frac{(3-m^2)}{2}=\frac{11.52*6.3}{3}\,\frac{(3-0.8^2)}{2}*2=57\,KN/m$$

* Torsion Reinforcement Must put for every Corner.

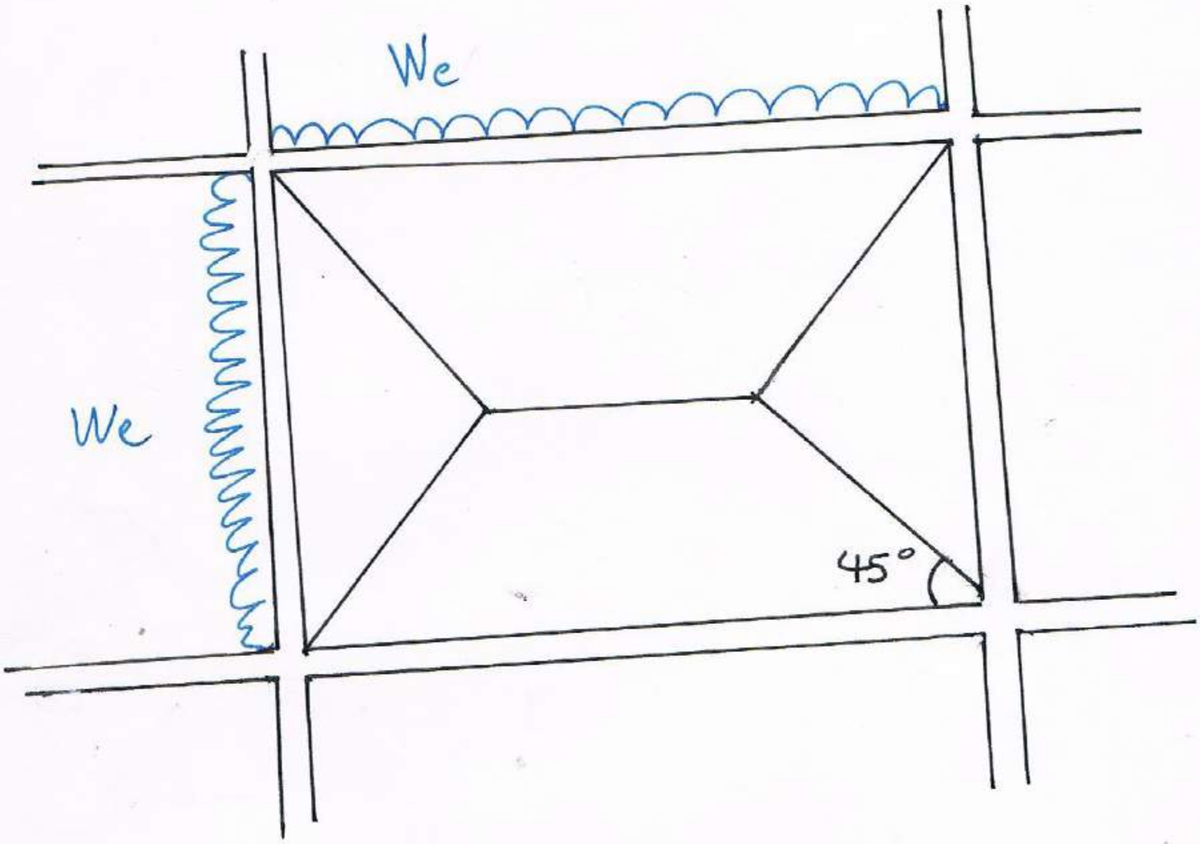


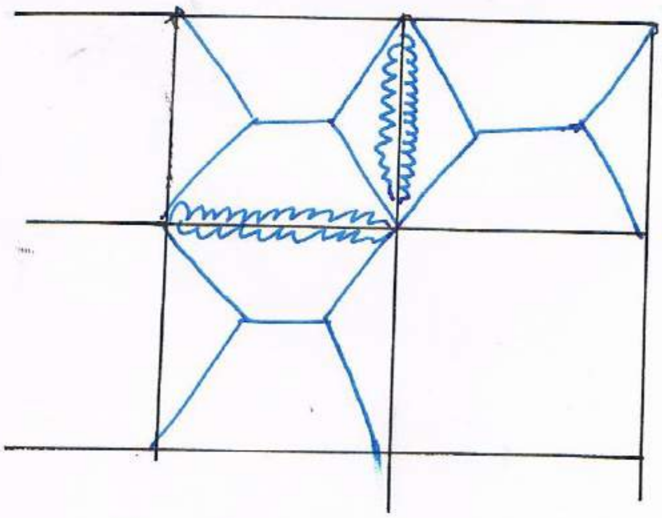
Ø10/360
Ø8/360
Ø8/300
Ø10/300
Ø10/300
Ø10/300
0.9 m
1.5m
Ø 10/360
1.8m
Ø8/360
1.5 m
2m
Ø10/360
Ø 10/360
1.8m
2.5m
1.8m
2.5m
1m
1.8m
Bot
Top
Ø10/360 for each layer

W equivelant

$$W_e(\text{for short beams}) = \frac{W_u S}{3}$$

$$W_e(\text{for Long beams}) = \frac{W_u S}{3}\left(\frac{3-m^2}{2}\right)$$





Torsion Reinforcement:-

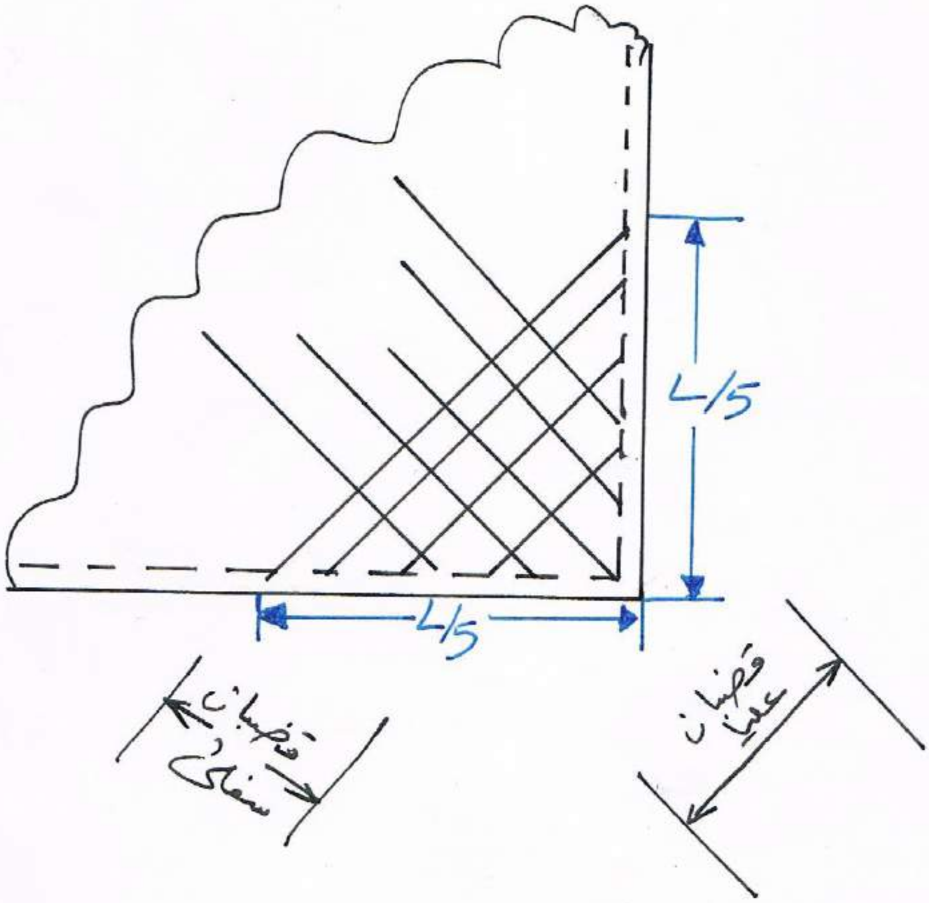
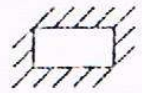
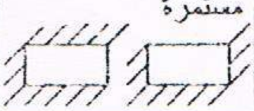
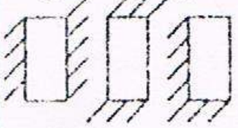
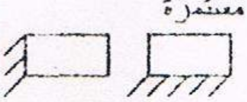


Table .(1) ١٥

نوع البلاطة	العزوم	الاتجاه القصير – نسبة العرض الى الطول (m) 1.0	0.9	0.8	0.7	0.6	0.5	الاتجـــاه الطويل لجميع قيم (m)
بلاطة داخلية 	$M_{\bar{u}cont}$	0.033	0.040	0.048	0.055	0.063	0.083	0.033
	$M_{\bar{u}disc}$	———	———	———	———	———	———	———
	M_u^+	0.025	0.030	0.036	0.041	0.047	0.062	0.025
أحد الحافات غير مستمرة 	$M_{\bar{u}cont}$	0.041	0.048	0.055	0.062	0.069	0.085	0.041
	$M_{\bar{u}disc}$	0.021	0.024	0.027	0.031	0.035	0.042	0.021
	M_u^+	0.031	0.036	0.041	0.047	0.052	0.064	0.031
حافتان غير مستمرة 	$M_{\bar{u}cont}$	0.049	0.057	0.064	0.071	0.078	0.09	0.049
	$M_{\bar{u}disc}$	0.025	0.028	0.032	0.036	0.039	0.045	0.025
	M_u^+	0.037	0.043	0.048	0.054	0.059	0.068	0.037
ثلاث حافات غير مستمرة 	$M_{\bar{u}cont}$	0.058	0.066	0.074	0.082	0.090	0.098	0.058
	$M_{\bar{u}disc}$	0.029	0.033	0.037	0.041	0.045	0.049	0.029
	M_u^+	0.044	0.050	0.056	0.062	0.068	0.074	0.044
جميع الحافات غير مستمرة 	$M_{\bar{u}cont}$	———	———	———	———	———	———	———
	$M_{\bar{u}disc}$	0.033	0.038	0.043	0.047	0.053	0.055	0.033
	M_u^+	0.050	0.057	0.064	0.072	0.080	0.083	0.050

ملاحظات:

1- النهاية الوشرة يقصد بها مستمرة

2- الرموز $M_{\bar{u}cont}$ = العزم السالب للنهاية المستمرة .

$M_{\bar{u}disc}$ = العزم السالب للنهاية غير المستمرة

M_u^+ = العزم الموجب .

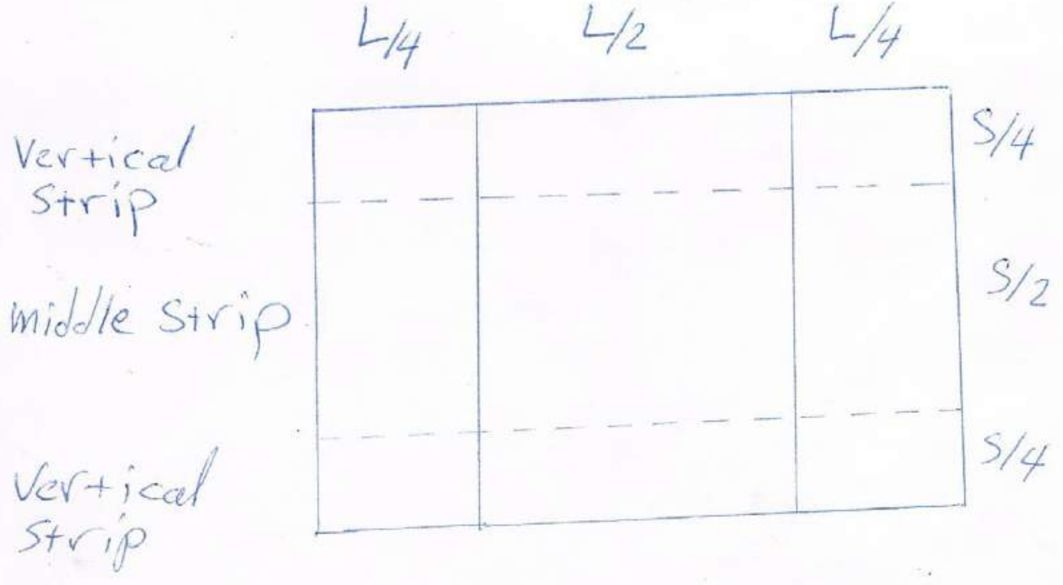
Design of Two-Way Slabs :-

* If $\frac{L}{S} < 2.0$, The slab Should be desinged as two-way slab.
* There are many Kinds of two-way slab like:-

1- Slabs supported by shallow beams.
2- Flat plate Slabs.
3- Flat Slabs :-
   a- Flat slabs with drop Panels.
   b- Flat slab with column capital.
   c- Slab with drop panel & capital.
4- Tow way ribbed slabs.

Design of Edge Supported Slabs :-

* There are three method for design this type of slab
* We will use the second method for the design.



- * ρ must be equal or less than ρ_{max}

$$\rho \leq \rho_{max}$$

if not, then the thickness must be increased.

* $A_{s\,min}$ must equal to area of steel for temperature & shrinkage.

- Calculations of Posative & Negative Moments in both directions.

$$M_u = C\, W_u\, S^2.$$

C: Coeffecint, W_u: Ultimate load, S: short span length

$$\text{Bending Moment for Vertical Strip} = \frac{2}{3} * \text{Bending Moment for middle Strip}$$

- Steel Reinforcement for middle Strip. :-

Finde ρ & it must equal or less than ρ_{max}

* See table (13)

- Distribution of Steel Reinforcement :-

$$S = \frac{1000}{A_s/A_b} = \frac{1000}{N} \quad , \quad S \leq 2h$$

In same places we mus add bars ($A_{s\,add}$).

Procedure of Design :-

- Thickness of slab.

$$h \geq \begin{cases} \frac{P}{180} \\ 90\,mm \end{cases}$$

محيط البلاطة

- $W_u = 1.2D + 1.6L$

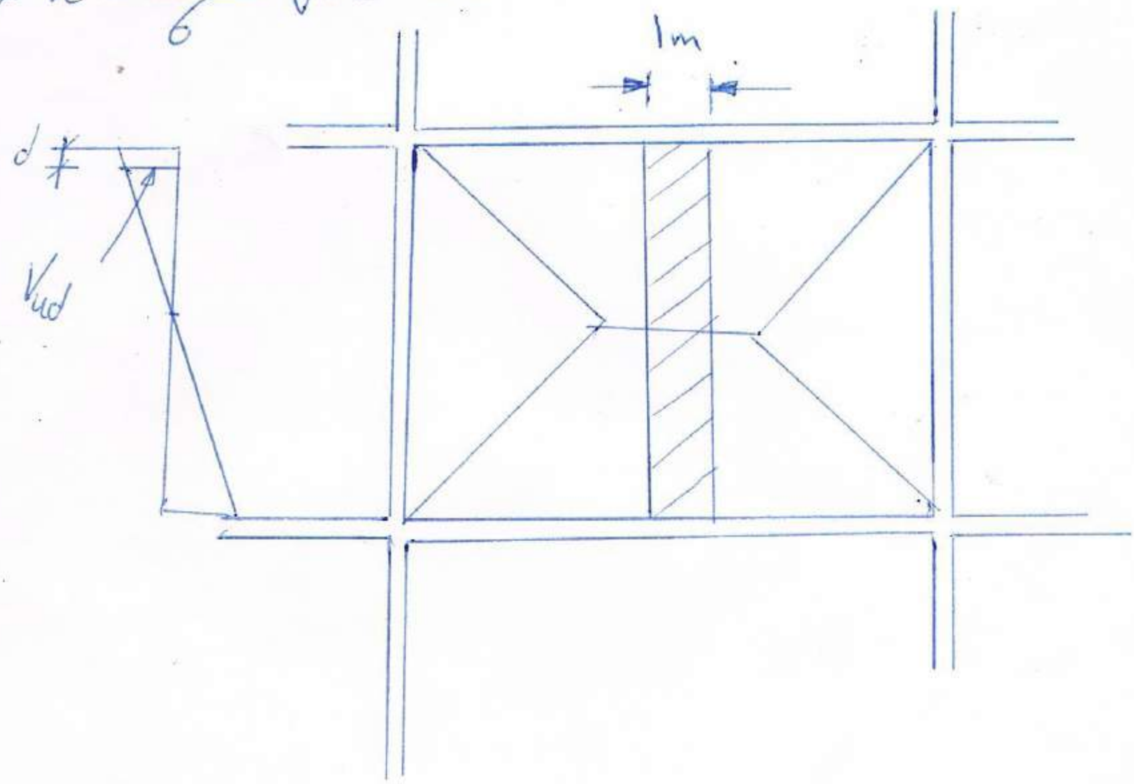
- Check of slab thickness according to shear requirements

$$V_{ud} = W_u\left(\frac{S_n}{2} - d\right)$$

where S_n – Short Span

Comparing between Shear force with Shear strength design which is calculated as follow:-

$$\phi V_c = \frac{0.75}{6}\sqrt{f_c'}\,bd$$



Then $V_{ud} \leq \phi V_c$

- Distribution of loads on beams :-

* for short beams $W_e = \frac{W_u S}{3}$

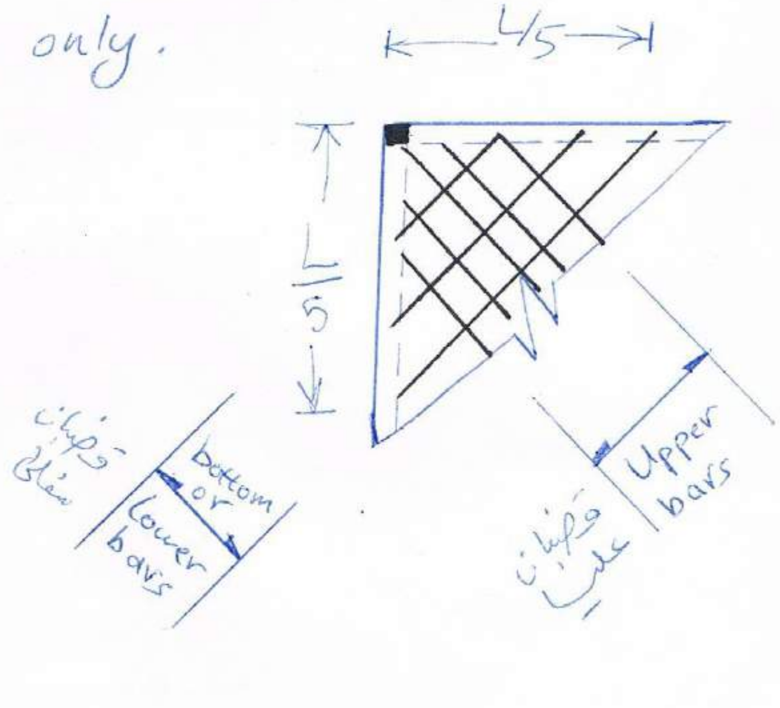
* for long beams $W_e = \frac{W_u S}{3}\left(\frac{3-m^2}{2}\right)$

when W_e is equivelant uniformly distrebuted load.

W_u: Factord load for one sequared meter of slab area.

- Torsion Reinforcement :-

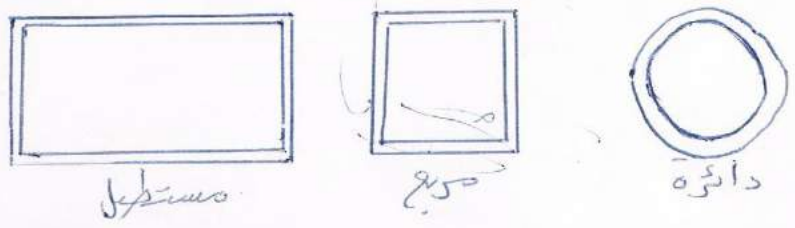
This reinforcement is added for extiror corner (edge) only.



Tanks on Ground :-

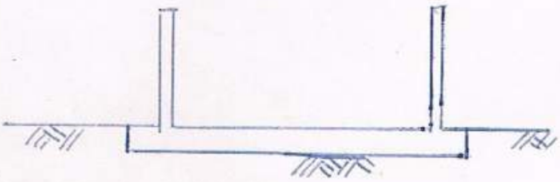
Sequre, rectangular or circular (closed or open)

uses: water reservoirs, settling tanks .... etc.

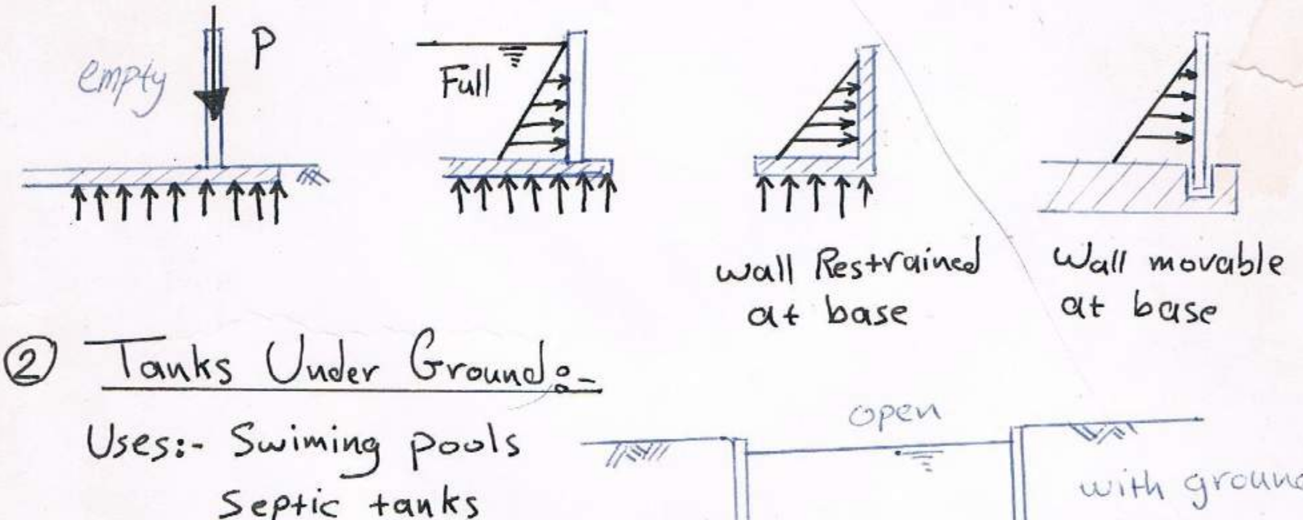


Plan

Section



Loading: subjected to internal pressure only

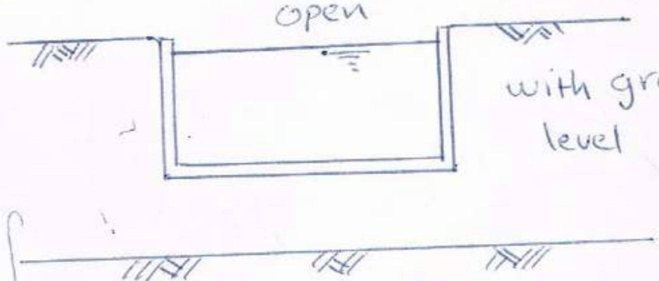


wall Restrained at base

Wall movable at base

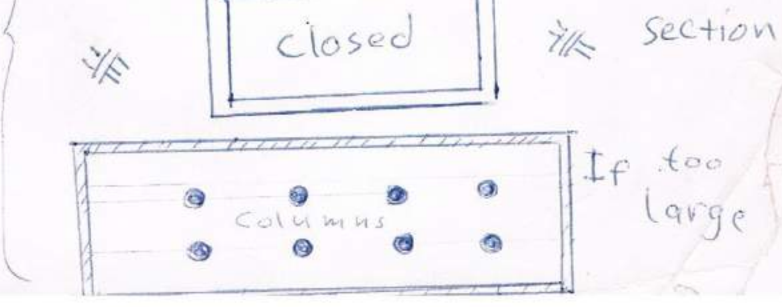
② Tanks Under Ground :-

Uses:- Swiming pools
- Septic tanks
- Reservoirs
- gas holder
- Purifications tanks
- etc...

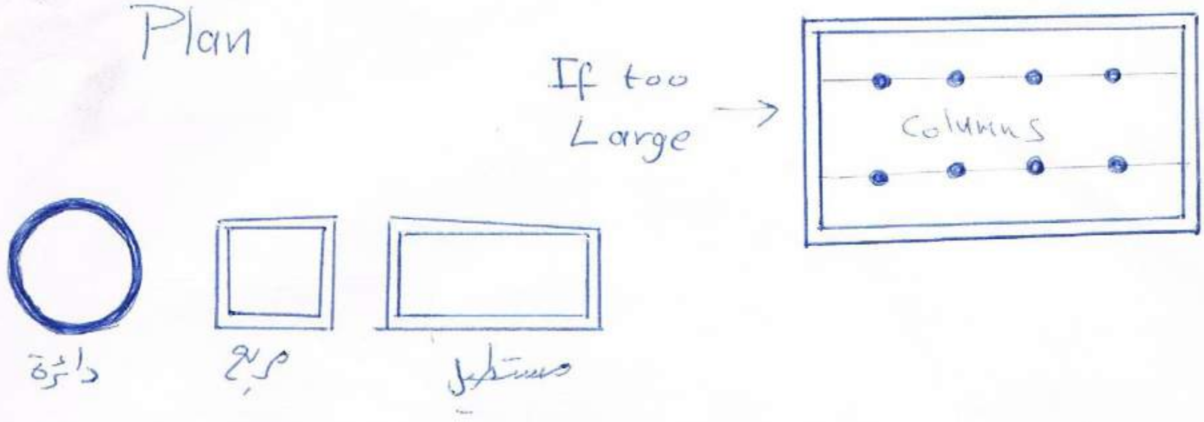


with ground level

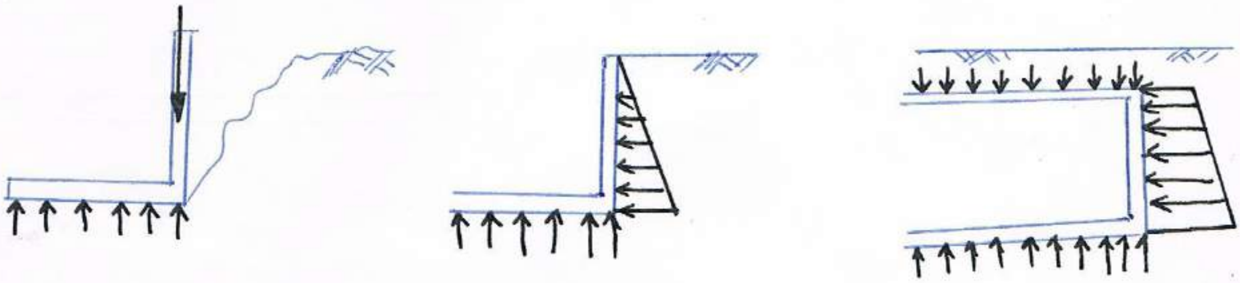
sections



Plan

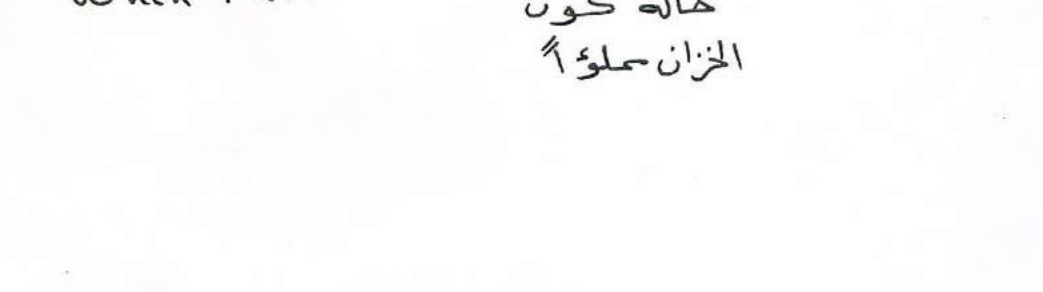


Loading :- Subjected to both internal & external pressure.



When empty before back filling

عندما يكون الخزان فارغاً



When Full

حالة كون الخزان مملوءاً

d- Base Slab :-

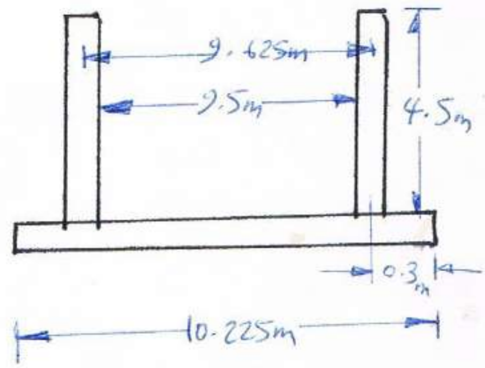
When the tank is empty :-

W_w = load or weight of the walls $= \pi(9.5 + 0.125) * 0.125 * 4.5 * 24$

$= 408.2\ kN$

q = Intensity of soil pressure below the base slab

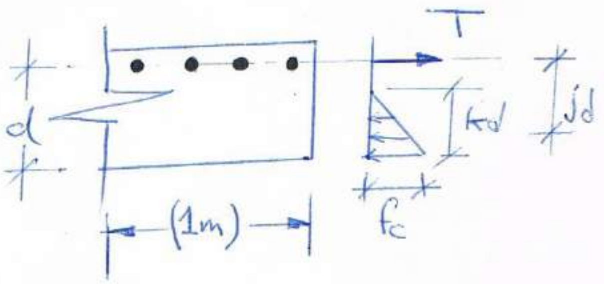
$$q = \frac{W_w}{\frac{\pi}{4}(10.225)^2} = \frac{408.2}{\frac{\pi}{4}(10.225)^2} = 4.97\ kN/m^2$$



Max B.M. at the center of slab $= \frac{qD^2}{16}$ (simply supported)

أن الجزء الناتئ من البلاطة تقلل من العزم السالب لكن هنا لا يؤخذ بنظر الاعتبار وذلك في الجانب الآمن، عليه نأخذ الفضاء الصافي $S = 9.5m$

$$B.M. = \frac{4.97(9.5)^2}{16} = 28\ kN.m/m$$



$$k = \frac{n}{n + r} = \frac{15}{15 + \frac{80}{8}} = 0.6$$

$$J = 1 - \frac{k}{3} = 0.8$$

$$M = 0.5 f_c\, k d\, b\, j d = 0.5\, f_c\, k\, j\, b d^2$$

$$d = \sqrt{\frac{28 \times 10^6}{0.5 * 8 * 0.6 * 0.8 * 1000}} = 120\ mm$$

Thickness of the slab = 120 + 40 + 30 = 190 mm (cover / $1.5 * \phi$)

Slab reinforcement :-

$f_{st} = 80\ MPa$, $J = 0.8$, $A_s = \frac{M}{f_s\, j d} = \frac{28 * 10^6}{80 * 0.8 * 120} = 2500\ mm^2/m$

Use ϕ 20mm $A_b = 314\ mm^2$ $N = \frac{2500}{314}$

$$S = \frac{1000}{2500/314} = 125.6\,mm$$

Use ϕ 20mm @ 125 mm c/c in both directions.

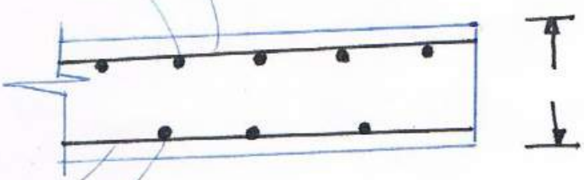
This Steel must be provided at top face of slab.

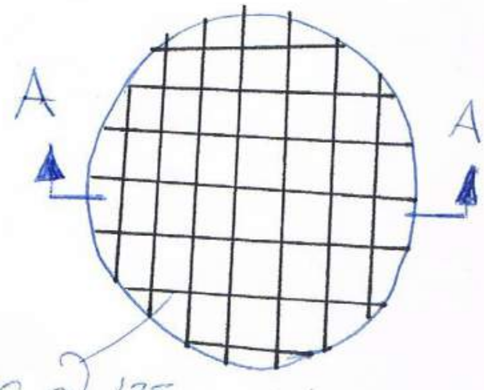
Provide secondary bars @ bottom 0.25%

$$A_s = \frac{0.25}{100} * 1000 * 190 = 475\ mm^2/m$$

$$S = \frac{1000}{475/79} = 166\,mm$$

Use ϕ 10mm @ 160mm c/c bars @ bottom in both direction





e:- Bearing Capacity (check):-

assume $q_{all} = 100\ kN/m^2$

when tank is full

W_T = wt of wall + wt of slab + wt of water

$$= 408.2 + 0.190 \times 24 \times \frac{\pi}{4}(10.225)^2 + 10 \times 4.5 \times \frac{\pi}{4}(9.5)^2 = 3972.34\ kN$$

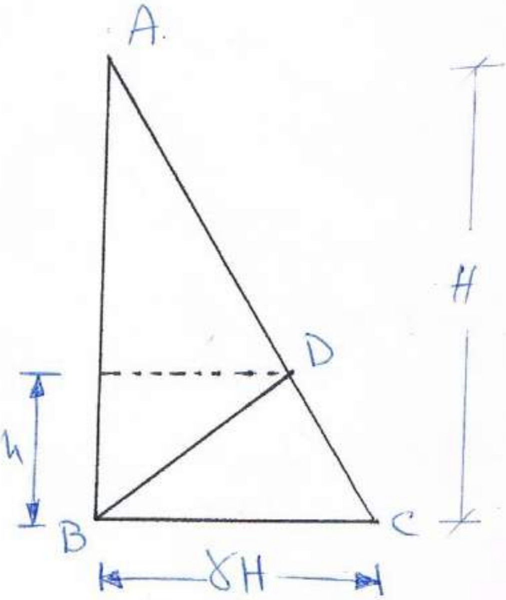
$$\text{Max Pressure on soil} = \frac{3972.34}{Area} = \frac{3972.34}{\frac{\pi}{4}(10.225)^2} = 48.38\ kN/m^2 < 100\ kN/m^2$$

$\therefore$ ok

Circular Tank with Fixed Base:-

In this method some portion of the tank at the base acts as a cantilever & some load at the bottom is taken by the cantilever effect.

The cantilever effect depends on the dimensions of tank & the thickness of wall.



i- for $\frac{H^2}{D.t}$ is between 6 to 12 the value of $h = \frac{H}{3}$ or (1m) which ever is more.

ii- for $\frac{H^2}{D.t}$ is between (12 to 30) the value $h = \frac{H}{4}$ or (1m) whichever is more.

Portion ABD is taken as Pressure causing hoop tension & DBC is taken as cantilever load.

The max. hoop tension occurs at D.

Example :- $H = 3.65m$, $D = 11.3m$, $t = 0.16m$

$$\frac{H^2}{D.t} = \frac{3.65^2}{11.3 \times 0.16} = 7.369 > 6$$

$$\therefore h = \frac{H}{3} = \frac{3.65}{3} = 1.22 > 1m$$

hence $h = 1.22m$

Calculation for Reinforcement where the Hoop Force is Max:-

$T = \frac{PD}{2} = \frac{10\ (3.65 - 1.22)}{2} * 11.3 = 137.3\ kN$

$A_s = \frac{T}{f_s} = \frac{137.3 * 10^3}{80.0} = 1716.25\ mm^2$ use ϕ 12 mm

$S = \frac{1000}{(1716.25 / \frac{\pi}{4} * 12^2)} = 65.89\ mm\ c/c$

Use S = 130 mm c/c on both faces

Calculation of Max. B.M. :-

$B.M._{max} = \frac{1}{2} \gamma H \cdot h \cdot \frac{h}{3} = 9\ kN \cdot m$

$M = A_s f_s \cdot jd \Longrightarrow d_{eff} = t - (cover + \frac{\phi}{2})$

$d = 160 - (40 + \frac{12}{2}) = 114\ mm$

$A_s = \frac{M}{f_s jd} = \frac{9 * 10^6}{80 * 0.85 * 114} = 1160.1 \simeq 1160\ mm^2$

Provide 12 ϕ @ 97 mm c/c USE ϕ12 @ 90 mm c/c

Nominal vertical reinforcement:

$\frac{0.25}{100} * 160 * 1000 = 400\ mm^2$

Provide $\frac{1000}{(400 / \frac{\pi}{4} * 12^2)} = 282.74\ mm\ c/c$

Use 560 mm c/c in each face.

Area of steel

$$A_s = \frac{T}{f_s} = \frac{155.65 \times 10^3}{80} = 1945.625\ mm^2$$

Use ϕ 12mm $A_b = 113\ mm^2$

$$S = \frac{1000}{1946/113} = 58 \simeq 60\ mm\ c/c$$

for both faces use $S = 120\ mm\ c/c$

Design for Cantilever Action:-

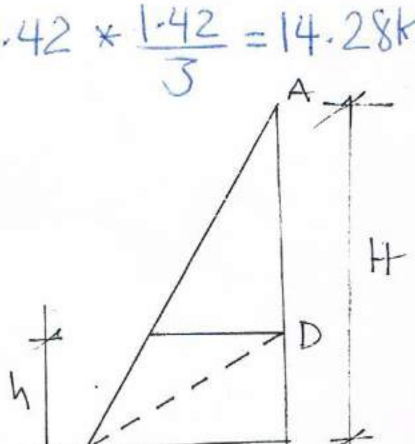
$$M_{max} = \frac{1}{2}\gamma H \cdot h \cdot \frac{h}{3} = \frac{1}{2} * 10 * 4.25 * 1.42 * \frac{1.42}{3} = 14.28\ kN.m/m$$

$$M = \frac{1}{2} f_c k d b j d$$

$$k = \frac{n}{n+r} = \frac{9}{9+\frac{80}{9}} = 0.5$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.5}{3} = 0.83$$

$$d = \left(\frac{2 * 14.28 * 10^6}{9 * 0.5 * 1000 * 0.83}\right)^{\frac{1}{2}} = 87.44\ mm$$



Thickness of the wall $t = 87.44 + 40 + \frac{12}{2} = 133.44\ mm < 185\ mm$ = OK

$$M = A_s f_s j d \Longrightarrow A_s = \frac{M}{f_s j d} = \frac{14.28 * 10^6}{80 * 0.83 * (185 - (40+6))} = 1547\ mm^2/m$$

$$S = \frac{1000}{1547/113} = 73\ mm\ c/c$$

use ϕ12mm @ 70mm c/c

nominal Vertical Reinforcement = (0.2 - 0.3)% of gross concrete area:-

Use 0.3% $\Rightarrow A_{s_{secondary}} = \frac{0.3}{100} * 185 * 1000 = 555 \ mm^2/m'$

for two faces $A_s = \frac{555}{2} = 277.5 \ mm^2/m'$

$S = \frac{1000}{277.5/113} = 407.2 \Rightarrow$ use $S = 400$ mm c/c for both faces.

Design of Base Slab :-

Critical case when tank is empty

Load from wall $= \pi (11 + 0.185) * 4.5 * 0.185 * 24$
$= 644.28$ kN

Intensity of soil Pressure $= \frac{644.28}{\frac{\pi}{4} * (12.34)^2} = 5.39 \ kN/m^2$

Max. B.M. $= \frac{3}{16} q$

E.X :- Design a circular tank with fixed base, for a capacity of $400 m^3$. The depth of water is to be 4.5m, including a free-board of 25cm. $f_c = 9 N/mm^2$, $f_{ct} = 1.2 N/mm^2$, $n = 9$, $f_s = 80 N/mm^2$. Bearing Capacity of soil $= 70 \; kN/m^2$.

Solution :- effective depth $= 4.5 - 0.25 = 4.25m$

$$V = \frac{\pi}{4} D^2 * 4.25 = 400 m^3$$

$D = 10.95$ Use $D = 11.0m$

$t_{min} = ?$ $\quad T = \frac{1}{2} \gamma h D = \frac{1}{2} \times 10 * 4.5 * 11 = 247 \; kN/m'$

$$A_{st} = \frac{T}{f_{st}} = \frac{247 \times 1000}{80} \simeq 3094 mm^2$$

$$t_{min} = \frac{1}{1000}\left[\frac{T}{f_{ct}} - (n-1) A_{st}\right] \; ; \; n = \frac{E_s}{E_c} = \frac{200 \times 10^3}{4700\sqrt{f_c'}}$$ اذا لم تعطى n

$$= \frac{1}{1000}\left[\frac{247 * 1000}{1.2} - (9-1) * 3094\right]$$

$= 181 mm$ use $t = 185 mm$

$$\frac{H^2}{Dt} = \frac{(4.25)^2}{11 * 0.185} = 8.876 \quad \text{(between 6-12)}$$

$$\therefore \frac{4.250}{3} = 1.4166 > 1m$$

$\therefore h = 1.416m$

Design of Walls for hoop tension :-

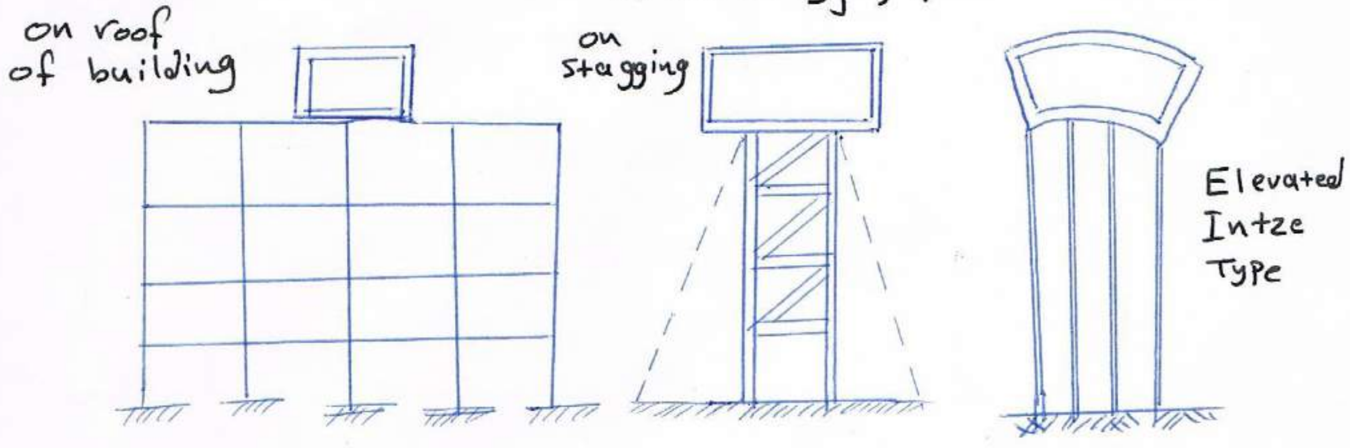
$$\text{Max. hoop tension} = \frac{\gamma (H-h) D}{2} = \frac{10(4.25 - 1.42) * 11}{2} = 155.65 \; kN$$

③ Tanks above Ground :- (Elevated Tanks)

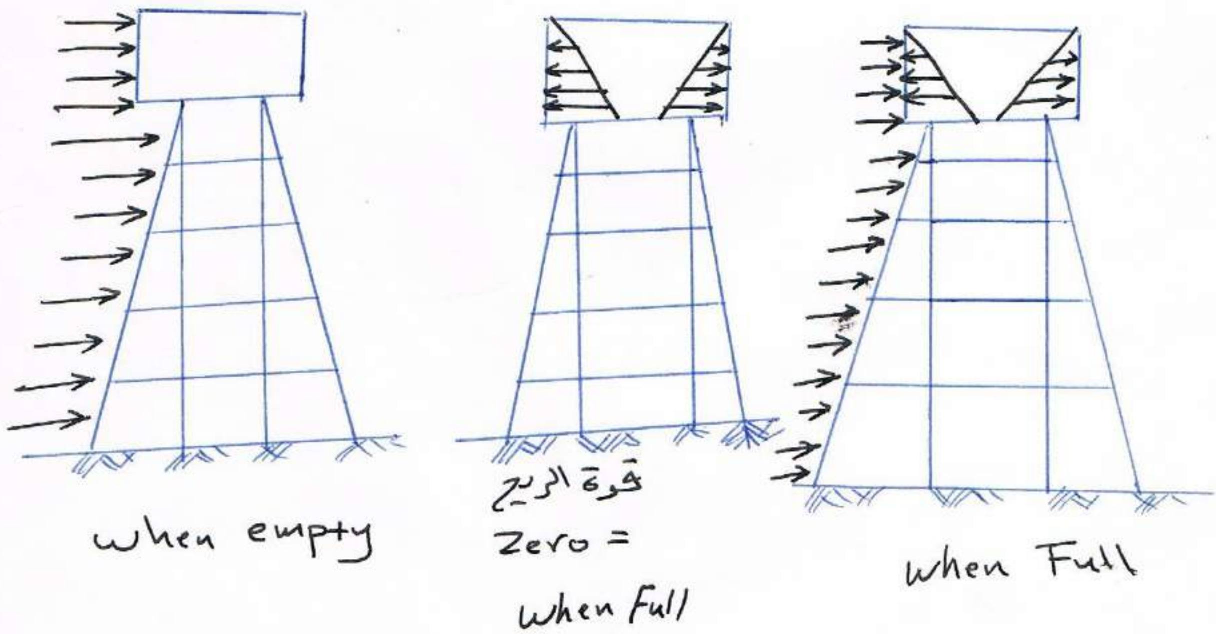
Usually Closed

Sequare, rectangular, Circular or Intze type :-

Uses : Water Tanks
- Tanks on roof of building
- Tanks on stagging (columns & beams)



Loading :- Internal pressure & external pressure (Wind Loading all direction)



Use main reinforcement at different heights.

$$T = \frac{1}{2}\gamma D h = \frac{1}{2} * 10 * 9.5 * h = 47.5h\ (kN) = 47500\ h\ (N)/m'$$

$$A_{st} = \frac{T}{80}$$

Devide the height in two strips (Rings), 1m each

depth h(m)	T=47500h(N)	$A_{st}=\frac{T}{80}$ (mm²)	ø	Vertical spacing	if on two faces
4.5 - 3.5	213 750	2672	ø16	75mm	150mm
3.5 - 2.5	166 250	2078	ø16	90mm	180mm
2.5 - 1.5	118 750	1484	ø16	130 mm	260mm
1.5 - 0	71 250	891	ø16	225mm	450mm

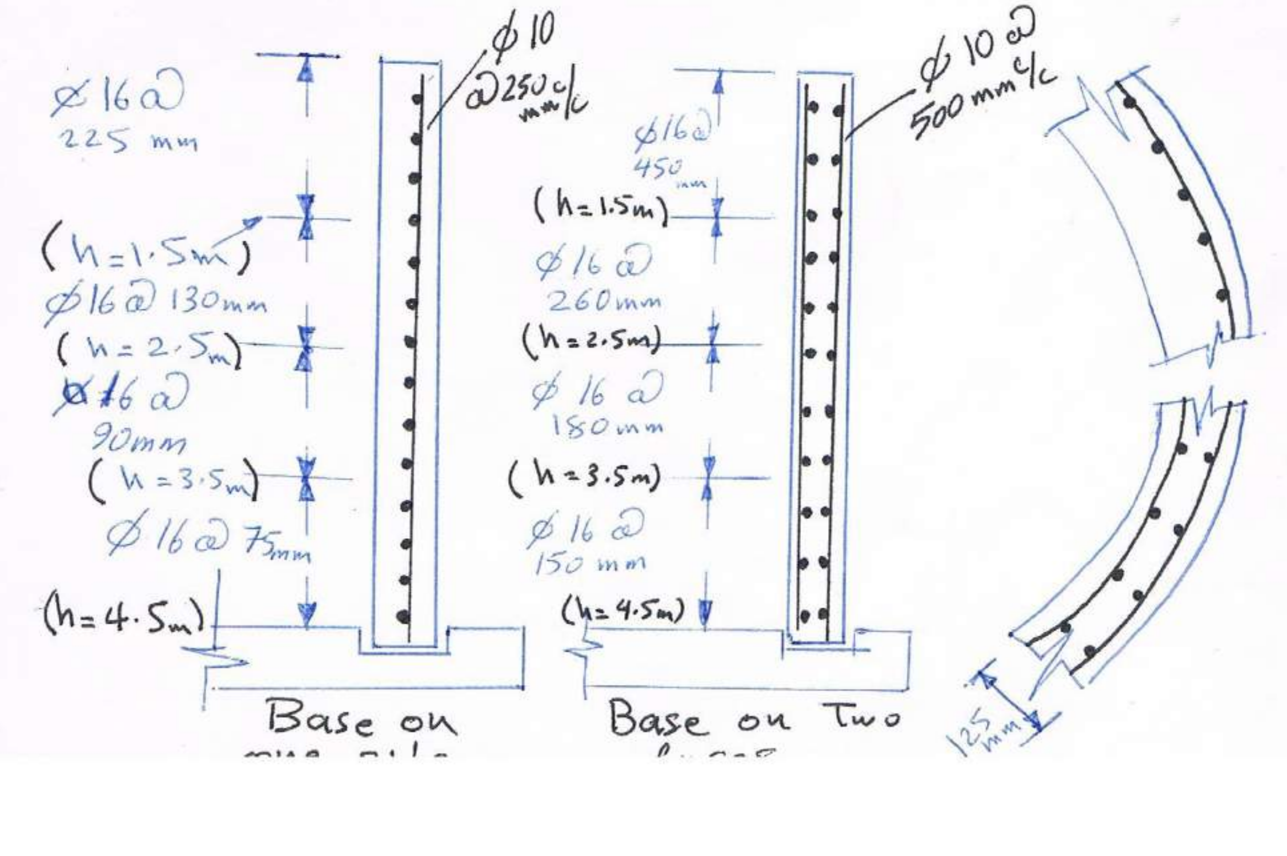
$A_b = 201 mm^2$, $N = \frac{A_{st}}{201}$, $S = \frac{1000}{N} = \frac{201 * 10^3}{A_{st}}$

C:- Secondary Reinforcement :-

Provide $0.25\% = \frac{0.25}{100} * 1000 * 125 = 312\ mm^2/m'$

Use ø10mm $A_b = 79$ mm $N = \frac{312}{79}$, $S = \frac{1000}{312/79} = 253$ mm

Use ø10mm @ 250mm c.c.



Example:- Design a cylindrical water tank for 300,000 liter capacity, assume the wall is not monolithic with the base. Given $f_{st} = 80 MPa$, $f_{ct} = 1.38 MPa$, $n = 15$, $f_c = 8$.

Solution:- $1000\ litre = 1 m^3$

$\therefore$ Required Capacity $= 300 m^3$

* take height of the tank $= 4.5 m$

* Capacity = Volume of water $= \frac{\pi D^2}{4} * h = 300 m^3$

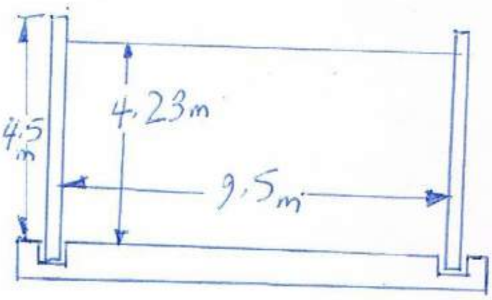
$\therefore D = \sqrt{\frac{4 * 300}{\pi * 4.5}} = 9.2 m$

$\therefore$ Take the diameter of tank (inside) $= 9.5 m$

Then capacity $= \frac{\pi}{4} D^2 * h = \frac{\pi (9.5)^2 * (4.5)}{4} = 318.97 m^3 > 300 m^3$ o.k.

$D = 9.5 m$, $h = 4.5 m$

$h_{water} = \frac{4 * 300}{\pi (9.5)^2} = 4.23 m$



a. Reinforcement (Main):-

$T = \frac{1}{2} \gamma * D * h = \frac{1}{2} * 10 * 4.5 * 9.5 = 213.75 kN/m'$

$A_{st} = \frac{T}{f_{st}} = \frac{213.75 \times 1000}{80} = 2671.88 mm^2/m' \simeq 2672 mm^2/m'$

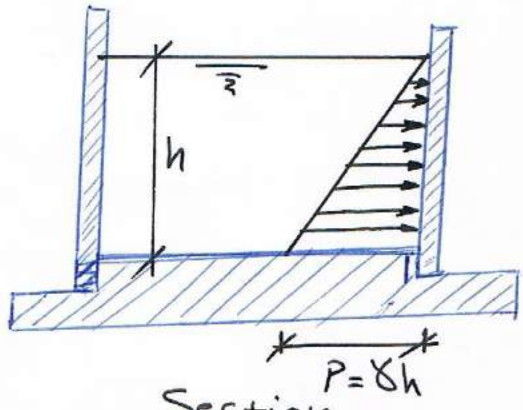
b. Thickness of Wall (t):-

$min\ t = \frac{1}{1000}\left[\frac{T}{f_{ct}} - (n-1)A_{st}\right] = \frac{1}{1000}\left[\frac{213.75 \times 10^3}{1.38} - (15-1) * 2672\right]$

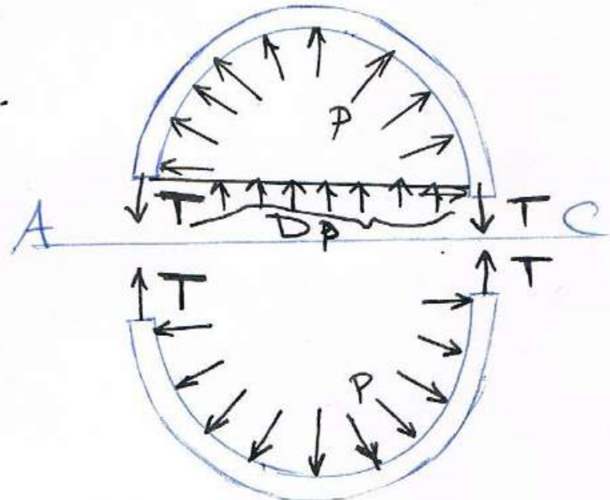
$= 117.48 mm \Rightarrow$ use $t = 125 mm$

Design of Circular Tanks :-

* In Designing water tanks, Working Design Method is used.
* Analysis & design of Sections Consider that the concrete is uncracked concrete, i.e. the tensile stresses in concrete is smaller than (f_r) to prevent lekage.



Section



a- Main Reinforcement :-

Total Pressure on Diameter

$$A_C = DP$$

$$2T = DP$$

$$\therefore T = \frac{DP}{2} = \frac{\gamma h D}{2} \text{ per meter of wall height}$$

γ = density of water = 10 kN/m^3

T = hoop tension in wall (circumferential tension)

Steel is required to resist this hoop tension

$$A_{st} = \frac{T}{\text{allowable steel stresses in tension}} = \frac{T}{f_{st}} \ (mm^2)$$

$$f_{st} \leq 80 \text{ MPa}$$ (in order to prevent lekage)

A_{st} = Steel per 1m height

يوضع الحديد الرئيسي على شكل حلقات على محيط الخزان في الوسط عند كون السمك
أو طبقتين واحدة إلى الخارج وأخرى إلى الداخل عند كون السمك يكفي لذلك.

b:- Wall thickness

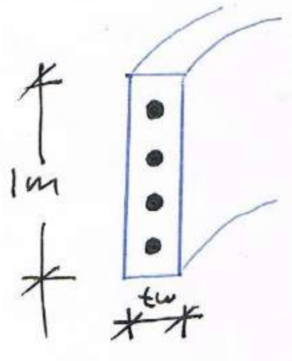
t_w = thickness of the wall

equivalant area = (area of 1m of wall)

$$A_{eq} = 1000\, t_w + (n-1) A_{st} \quad (mm^2)$$

This area must resist tensile stresses without cracking.

$$T = \{1000\, t_w + (n-1) A_{st}\} f_{ct}$$



f_{ct} = Tensile Stress in concrete

n = modular ratio $= \frac{E_s}{E_c}$

$$\therefore\ t_{min}\ required = \frac{1}{1000}\left[\frac{T}{f_{ct}} - (n-1) A_{st}\right] (mm)$$

Usually good quality concrete is used

[for example concrete mix (1:1.5:3) (cement: sand: gravel) is used to produce concrete use in tanks. This concrete give a Tensile strength $\geq$ 1.38 MPa

$n = 15$] but use $f_{ct} = 1.38$ MPa

min t = 100 mm but t_{min} = 125mm is preferd

or use t = 125mm for h = 0 to 6m

t = 150mm for h = 6m to 9m etc.

or t_{min} from above equation (choose whichever is greater)

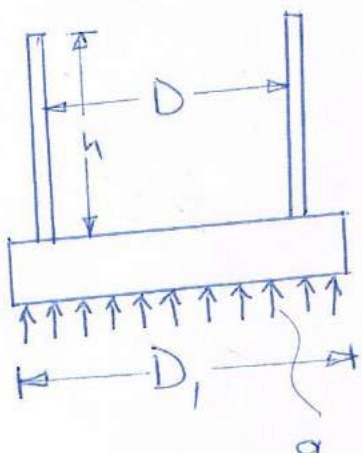
C: Secondary Reinforcement :-

Provide Secondary Reinf. (vertical) = 0.2 → 0.3% of wall area.

$$A_{s\,secondary} = \frac{0.25}{100} \times 1000 \times t_w \qquad \text{(for one meter)}$$

d: Base Slab:-

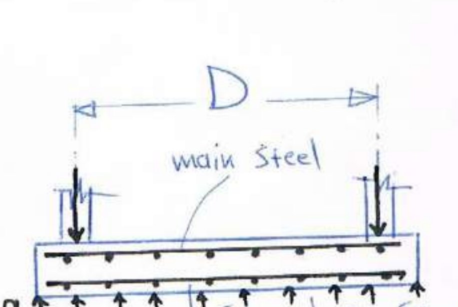
When the tank is empty the Earth Pressure due to dead loads of the walls Cause Negative moments (Tension on the top & compressive at the bottom of the slab). The's pressure can calculate as follow:-



$$W_t \text{ of side walls} = \pi \underbrace{(D+t)}_{m} \times t \times h \times 24 \quad (KN)$$

$$q = \text{upward pressure} = \frac{\text{weight}(W)}{\frac{\pi}{4} D^2}$$

$$\text{Max. B.M. at Center} = \frac{qD^2}{16}$$



The slab is considre is simply supported.

∵ In both sides the parts of tank is symetrical, the half of load is transmitted in all directions.

Min depth of base slab is 150 mm ⟶ 200 mm

Provide main Reinforcement for this B.M. at top in both directions & provide (0.2-0.3)% Secondary Reinf. in both directions at bottom of slab.

e- Bearing Capacity :-

Check the allowable bearing capacity of soil when tank is full of water.

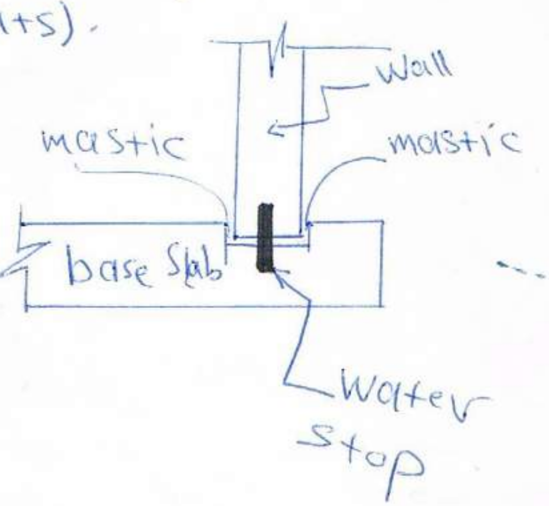
$$W_{water} + W_{walls} + W_{slab} = W$$

$$q_{net} = \frac{W}{\frac{\pi}{4} D_1^2} \leq q_{allowable}$$

If more increase the area of the base slab.

f- Other Requirements :-

Water stop (Rubber) must be provide at the joint between the base slab & the walls in all types of tanks. (and at the construction joints).



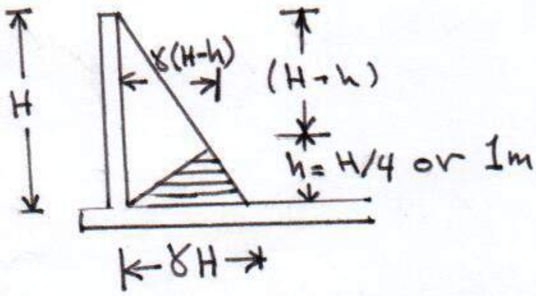
Dr.NAHLA N. HILAL

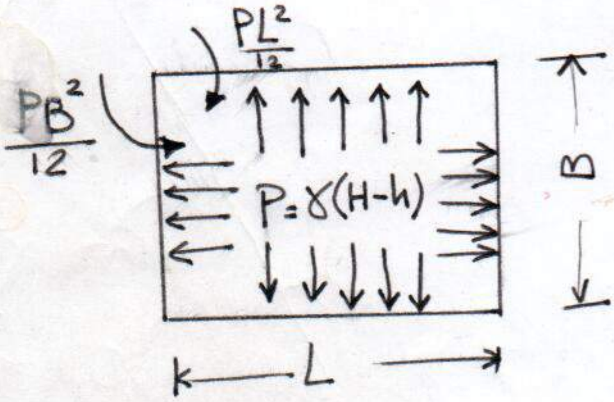
[redacted] of Rectangular Tanks:-

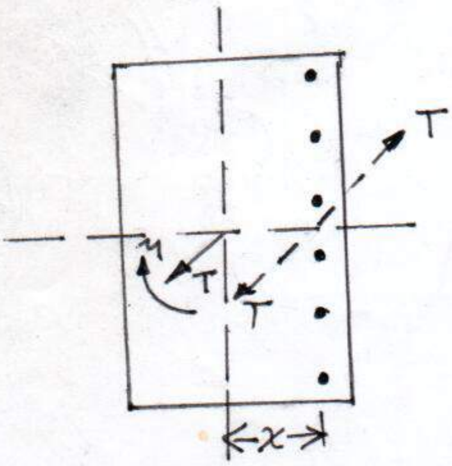
Approximate Design Method

(a) For tanks of ratio $(L/B) \leq 2$

Refering to the Fig. blow for the bottom height of $h = \frac{H}{4}$ or 1m (which ever is more), the bending is in vertical plane & this portion is designed as Cantilever.







The corners are designed for the maximum moment obtained after moment distribution with the intensity pressure $P = \gamma(H-h)$.

In the absence of moment distribution the bending moment may be computed by the following approximate expression:

Bending moment at centre of span $= \left(\frac{PB^2}{16}\right)$ (producing tension on outer face).

Bending moment at ends of span $= \left(\frac{PB^2}{12}\right)$ (producing tension on water face).

In addition to the bending moments, the walls are subjected to direct tension given by:-

Direct tension on long walls $= T_L = \gamma(H-h)\,B/2$
Direct tension on short walls $= T_b = \gamma(H-h)\,L/2$
Design moment $= (M - T.x)$

For B.M., $A_{st_1} = \left[\frac{M - Tx}{f_{st}.j.d}\right]$

$\therefore A_{st} = (A_{st_1} + A_{st_2})$

For direct tension $A_{st_2} = (T/f_{st})$

(b) Ratio of $(L/B) > 2$

In this case long walls are assumed to bend vertically & hence designed as cantilevers. Short walls are assumed to bend horizontally supported on long walls above H/4 or 1 m from bottom.

Bending moment for long walls $= \left(\frac{\gamma H^3}{6}\right)$

B.M. for short walls (above 1m from base) $= \frac{\gamma (H-h) B^2}{16}$

Maximum cantilever moment for short wall $= \left(\frac{\gamma H h . h}{2 * 3}\right)$

$= \left(\frac{\gamma H . h^2}{6}\right)$

In addition direct pulls are considered for long & short walls.

Example:

A rectangular Reinforced Concrete water tank with an open top is required to store 80,000 liters of water. The inside dimensions of tank as 6m x 4m. The tank rests on walls on all the four sides. Design the side walls of the tank for the following data.

Free ~~Bart~~ Board

$F.B. = 15cm$, $j = 0.84$, $n = 13$, $f_c = 7MPa$

$f_{st} = 125MPa$ (on faces away from water face), $f_{st} = 100MPa$ (on faces near water face)

Solution :-

* Height of water $= \left(\frac{80,000 * 10^3}{600 * 400}\right) = 335cm = 3.35m$

Height of side walls $= (335 + 15) = 350cm = 3.5m$

$(L/B) = 6/4 = 1.5 < 2$

$\therefore$ Walls designed as continous slab subjected to water pressure above $(H/4)$ or $(1m)$ from bottom.

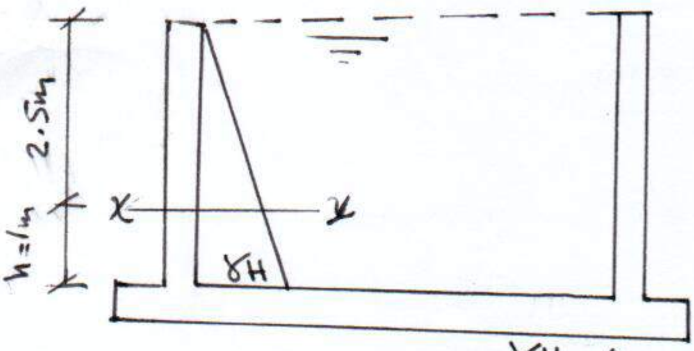
$\therefore P = \gamma(H-h) = (10 * 2.5) = 25 kN/m^2$

* Moments in Side walls:-

The moments in side walls is determined by moment distribution.

Fixed end moments:

Long Walls	Short Walls
$\left(\frac{P.L^2}{12}\right) = \left(\frac{25*6^2}{12}\right) = 75 kN.m$	$\left(\frac{PB^2}{12}\right) = \left(\frac{25*4^2}{12}\right) = 34 kN.m$
$\left(\frac{P.L^2}{8}\right) = \left(\frac{25 \times 6^2}{8}\right) = 112.5 kN.m$	$\left(\frac{PB^2}{8}\right) = \left(\frac{25*4^2}{8}\right) = 50 kN.m$



$\gamma H = (10 \times 3.5) = 35\ kN/m^2$

Tank Dimensions

0.6	0.4		0.4	0.6
+34	−75		+75	−34
+24.6	+16.4		−16.4	−24.6
+58.6	−58.6		+58.6	−58.6

$\frac{EI}{L}$, EI → متساوي للجدارين *

L مختلف *

Distribution Stiffness Factor

DF

for Long wall

$$DF_{\text{Long wall}} = \frac{k}{2k} = \frac{\frac{EI}{6}}{\frac{EI}{6} + \frac{EI}{4}} = \frac{EI}{EI}\left(\frac{\frac{1}{6}}{\frac{4+6}{24}}\right)$$

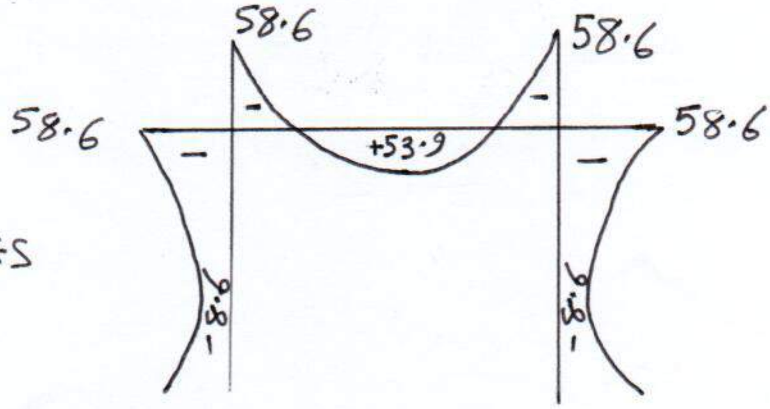
$$DF_{\text{Long wall}} = \frac{4}{10} = 0.4$$

$$DF_{\text{short wall}} = \frac{\frac{EI}{4}}{\frac{EI}{4} + \frac{EI}{6}} = \frac{\frac{1}{4}}{\frac{10}{24}} = \frac{6}{10} = 0.6$$

Moment at support = 58.6 kN·m

Moment at centre (Long walls) = (112.5 − 58.6) = 53.9 kN.m
Moment at centre (Short walls) = (50 − 58.6) = −8.6 kN.m

Bending Moments



* Design of Long & Short Walls :-

Maximum moment = 58.6 kN.m

$$d_{req.} = \sqrt{\frac{M_{max}}{0.5\,K\,j\,b\,f_c}} \quad , r = \frac{100}{7} = 14.286 \,, \; K = \frac{n}{n+r} = \frac{13}{13+14.3} = 0.476$$

$$d_{req.} = \sqrt{\frac{58.6 * 10^6}{0.5 * 0.476 * 0.84 * 1000 * 7}} = 204.63 \text{ mm}$$

(at xx)

Adopt overall depth = 250 mm

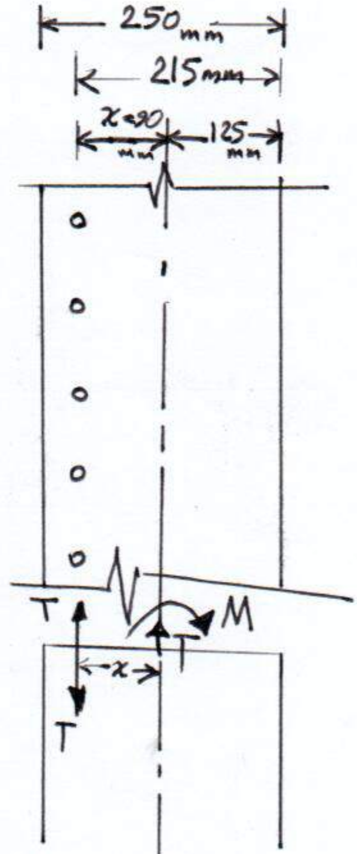
Use Effective depth = 215 mm

Direct tension in Long wall $= T = \left(\frac{P * B}{2}\right) = \left(\frac{25 * 4}{2}\right) = 50$ kN

Direct tension in short wall $= T = \left(\frac{P * L}{2}\right) = \left(\frac{25 * 6}{2}\right)$

$T = 75$ kN

$$A_{st}\,(\text{Long wall corners}) = \left[\frac{M - T * x}{f_{st} \cdot j \cdot d}\right] + \frac{T}{f_{st}}$$



Net Moment = $(M - T*x)$

T = Pull in Steel

Fig. (Moments in Cross Section)

Refering to the Fig. above

$$\therefore A_{st} = \left[\frac{(58.6*10^6)-(50*10^3*90)}{100*0.84*215}\right] + \left[\frac{50*10^3}{100}\right]$$

$$= 3495 \, mm^2$$

Use ϕ 20mm bar for reinforcement.

Spacing of 20mm ϕ bars $= \frac{1000}{3495/314} \simeq 90$ mm c/c

- Center of Span (Long Walls)

$$A_{st} = \left[\frac{(53.9*10^6-50*10^3*90)}{125*0.84*215}\right] + \left[\frac{50*10^3}{125}\right] = 2588.26 \, mm^2$$

Half the bars from inner face at support are bent twoards outer face at center providing an area of: $\frac{3495}{2} = 1748 \, mm^2$

For remaining area $(2588-1748) = 840 \, mm^2$

Provide ϕ 16mm with spacing $= \frac{1000}{840/201} = 239 \, mm\, c/c$

Use ϕ 16mm @ 200mm c/c

Short Walls :- (corners)

$$A_{st} = \left[\frac{(58.6*10^6)-(75*10^3*90)}{100*0.84*215}\right] + \left(\frac{75*10^3}{100}\right) = 3620 \, mm^2$$

Use ϕ 20mm with spacing $= \frac{1000}{\frac{3620}{314}} = 86 \, mm\, c/c$

Use ϕ 20mm @ 80 mm c/c (50% of bars bend twoards outer face at center)

At the center of short walls

$$A_{st} = \left[\frac{(-9*10^6)-(75*10^3*90)}{100*0.84*215}\right] + \left(\frac{75*10^3}{100}\right) = 122 \, mm^2$$

$$S = \frac{1000}{\frac{122}{314}} = 2573 \, mm\, c/c$$

∴ (The bars which bends from the corner is enough)

- Design for Cantilever Moment (For 1m height from bottom).

Cantilever moment $= (3.5*10*\frac{1}{2}*\frac{1}{3}) = 5.833 \, kN.m$

$$\therefore A_{st} = \left(\frac{5.833*10^6}{100*0.84*215}\right) = 323 \, mm^2$$

f_{st} j d

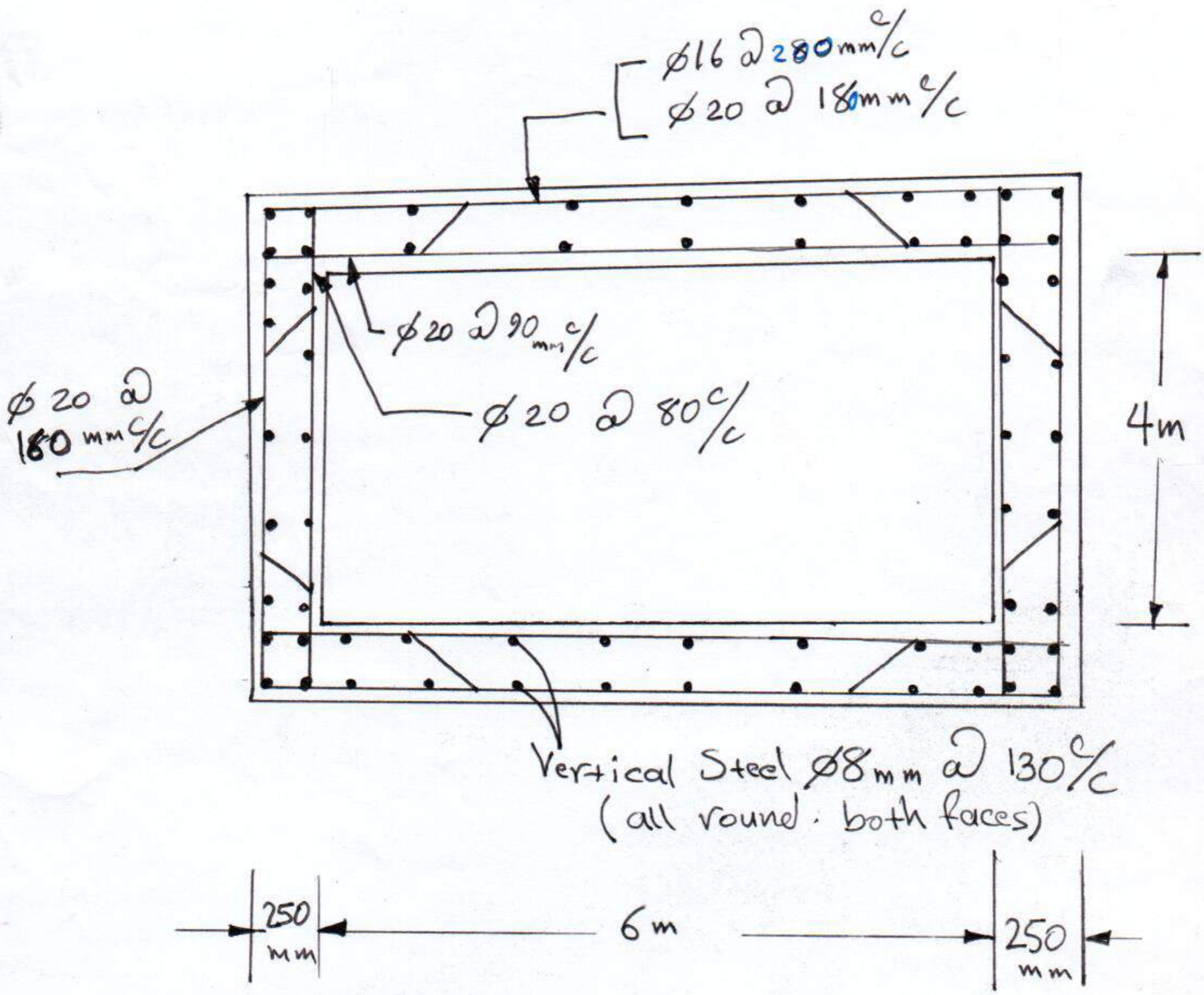
Minimum steel $= 0.3\% = \left(\frac{0.3}{100} * 1000 * 250\right) = 750 \text{ mm}^2$

Steel on each face $= \left(\frac{750}{2}\right) = 375 \text{ mm}^2$

Spacing of 8mm bars $= \left(\frac{1000}{\frac{375}{50}}\right) = 130 \text{ mm c/c}$

$A_b = \left(\frac{\pi}{4} * (8)^2\right) = 50 \text{ mm}^2$

Use ϕ 8mm c/c on both sides



Reinforcement Details in Water Tank (L/B < 2)